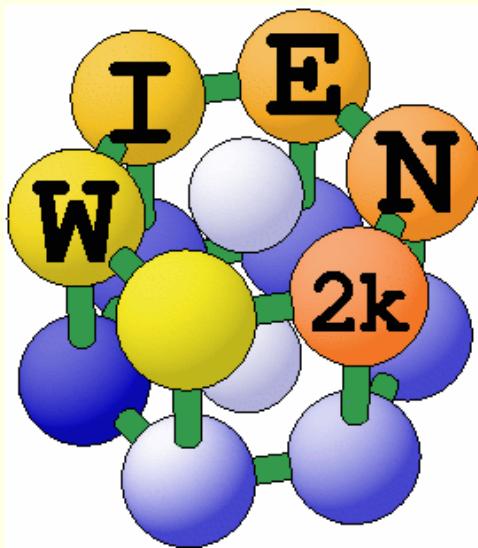
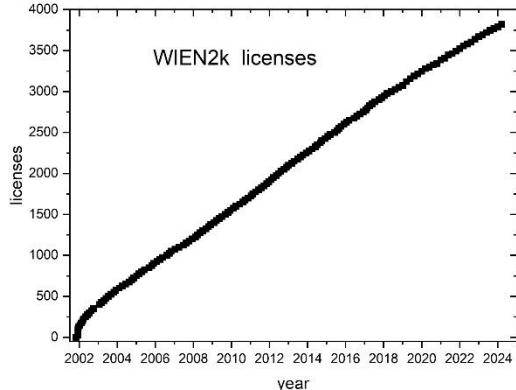


WIEN2k software package



WIEN97: ~500 users
WIEN2k: ~3900 users

An Augmented Plane Wave Plus Local Orbital Program for Calculating Crystal Properties

Peter Blaha
Karlheinz Schwarz
Georg Madsen
Dieter Kvasnicka
Joachim Luitz
Robert Laskowski
Fabien Tran
Laurence Marks

November 2001
Vienna, AUSTRIA
Vienna University of Technology

<http://www.wien2k.at>

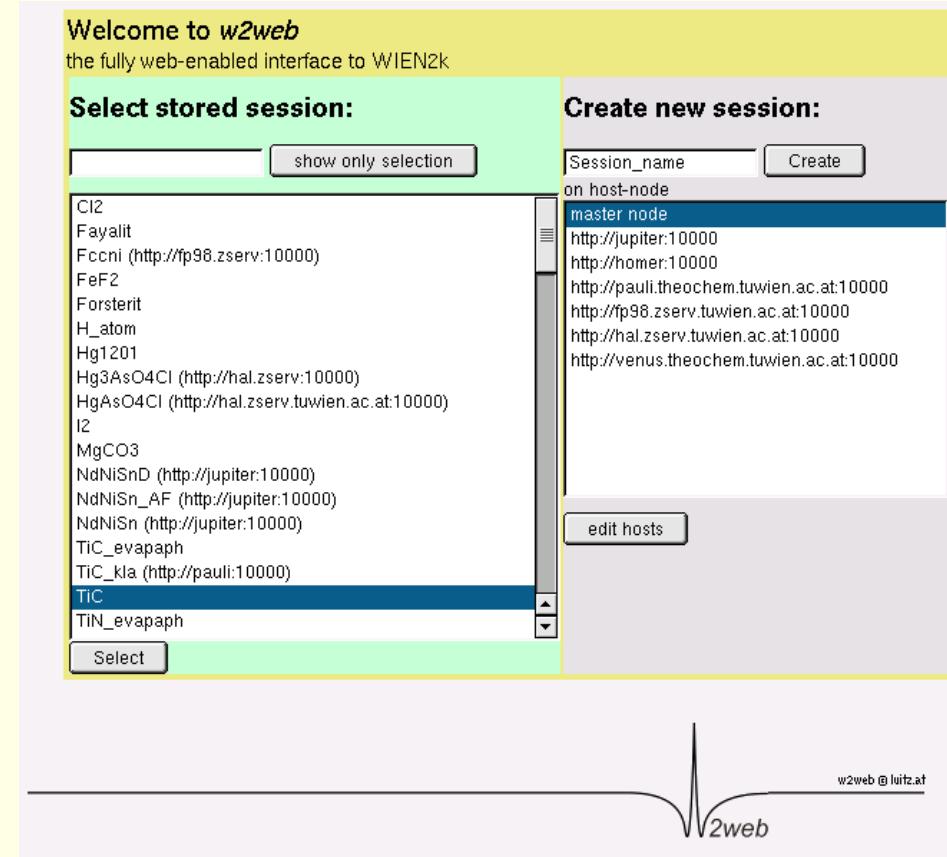
- WIEN2k consists of many independent F90 programs, which are linked together via C-shell scripts.
- Each „case“ runs in his own directory `./case`
- The „master input“ is called `case.struct`
- Initialize a calculation: `init_lapw`
- Run scf-cycle: `run_lapw (runsp_lapw)`
- You can run WIEN2k using any www-browser and the w2web interface, but also at the command line in an xterm.
- Input/output/scf files have endings as the corresponding programs:
 - `case.output1...lapw1; case.in2...lapw2; case.scf0...lapw0`
- Inputs are generated using STRUCTGEN(w2web) or makestruct and `init_lapw`

■ Based on www

- *WIEN2k can be managed remotely via w2web*

■ Important steps:

- *start w2web on all your hosts*
 - login to the desired host (ssh)
 - w2web (at first startup you will be asked for username/password, port-number, (master-)hostname. creates `~/w2web` directory)
- *use your browser and connect to the (master) host:portnumber*
 - firefox `http://fp98.zserv:10000`
- *create a new session on the desired host (or select an old one)*



The screenshot shows the "Welcome to w2web" interface. On the left, a list of stored sessions is displayed, including CI2, Fayalit, Fccni, FeF2, Forsterit, H_atom, Hg1201, Hg3AsO4Cl, HgAsO4Cl, I2, MgCO3, NdNiSnD, NdNiSn_AF, NdNiSn, TiC_evapaph, TiC_Kla, TiC, and TiN_evapaph. A "Select" button is at the bottom of this list. On the right, a "Create new session:" panel shows a table with columns for "Session_name" and "on host-node". It lists several entries, with "master node" selected, showing its details: "http://jupiter:10000", "http://homer:10000", "http://pauli.theochem.tuwien.ac.at:10000", "http://fp98.zserv.tuwien.ac.at:10000", "http://hal.zserv.tuwien.ac.at:10000", and "http://venus.theochem.tuwien.ac.at:10000". A "Create" button is in the top right of this panel. Below the table is an "edit hosts" button. At the bottom right of the interface is a small logo with the text "w2web @ luitz.at" and a stylized wave graphic.



w2web GUI (graphical user interface)

- **Structure generator**
 - *spacegroup selection*
 - *import cif or xyz file*
- **step by step initialization**
 - *symmetry detection*
 - *automatic input generation*
- **SCF calculations**
 - *Magnetism (spin-polarization)*
 - *Spin-orbit coupling*
 - *Forces (automatic geometry optimization)*
- **Guided Tasks**
 - *Energy band structure*
 - *DOS*
 - *Electron density*
 - *X-ray spectra*
 - *Optics*

Session: TiC
/area51/pbla/laapw/2005-june/TiC

StructGen™

You have to click "Save Structure" for changes to take effect!

Save Structure

Title: TiC

Lattice:

Type: F

Spacegroups from Bilbao Cryst Server

P
F
B
CXY
CYZ
CXZ
R
H
1_P1

Lattice parameters in Å

a=4.3280000386 b=4.3280000386 c=4.3280000386
α=90.000000 β=90.000000 γ=90.000000

Inequivalent Atoms: 2

Atom 1: Ti Z=22.0 RMT=2.0000 remove atom

Pos 1: x=0.00000000 y=0.00000000 z=0.00000000 remove
add position

Atom 2: C Z=6.0 RMT=1.9000 remove atom

Pos 1: x=0.50000000 y=0.50000000 z=0.50000000 remove
add position

Idea and realization by



Spacegroup P4₂/mnm

P4₂/mnm
D_{4h}¹⁴

No. 136

P 4₂/m 2₁/n 2/m

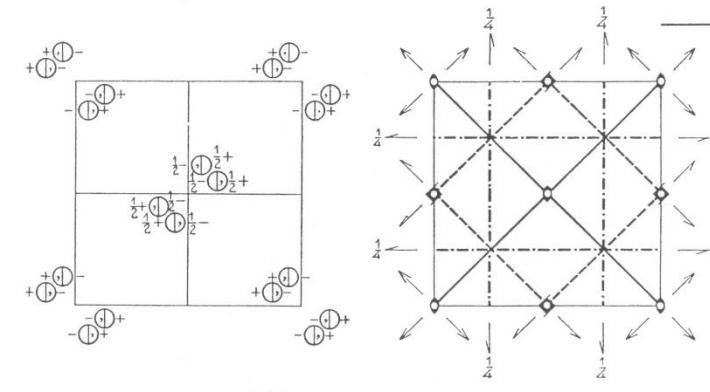
4/m m m Tetragonal

Structure given by:
spacegroup
lattice parameter
positions of atoms
(basis)

Rutile TiO₂:
P4₂/mnm (136)
a=8.68, c=5.59 bohr

Ti: (0,0,0)

O: (0.304,0.304,0)



Number of positions,
Wyckoff notation,
and point symmetry

Co-ordinates of equivalent positions

Conditions limiting
possible reflections

16	<i>k</i>	1	$x, y, z; \bar{x}, \bar{y}, z; \frac{1}{2} + x, \frac{1}{2} - y, \frac{1}{2} + z; \frac{1}{2} - x, \frac{1}{2} + y, \frac{1}{2} + z;$	$x, y, \bar{z}; \bar{x}, \bar{y}, \bar{z}; \frac{1}{2} + x, \frac{1}{2} - y, \frac{1}{2} - z; \frac{1}{2} - x, \frac{1}{2} + y, \frac{1}{2} - z;$	$y, x, z; \bar{y}, \bar{x}, z; \frac{1}{2} + y, \frac{1}{2} - x, \frac{1}{2} + z; \frac{1}{2} - y, \frac{1}{2} + x, \frac{1}{2} + z;$	$y, x, \bar{z}; \bar{y}, \bar{x}, \bar{z}; \frac{1}{2} + y, \frac{1}{2} - x, \frac{1}{2} - z; \frac{1}{2} - y, \frac{1}{2} + x, \frac{1}{2} - z.$
----	----------	---	---	---	---	---

General:
 hkl : No conditions
 $hk0$: No conditions
 $0kl$: $k+l=2n$
 $hh0$: No conditions

8	<i>j</i>	<i>m</i>	$x, x, z; \bar{x}, \bar{x}, z; \frac{1}{2} + x, \frac{1}{2} - x, \frac{1}{2} + z; \frac{1}{2} - x, \frac{1}{2} + x, \frac{1}{2} + z;$	$x, x, \bar{z}; \bar{x}, \bar{x}, \bar{z}; \frac{1}{2} + x, \frac{1}{2} - x, \frac{1}{2} - z; \frac{1}{2} - x, \frac{1}{2} + x, \frac{1}{2} - z.$
---	----------	----------	---	---

Special: as above, plus

} no extra conditions

8	<i>i</i>	<i>m</i>	$x, y, 0; \bar{x}, \bar{y}, 0; \frac{1}{2} + x, \frac{1}{2} - y, \frac{1}{2}; \frac{1}{2} - x, \frac{1}{2} + y, \frac{1}{2};$	$y, x, 0; \bar{y}, \bar{x}, 0; \frac{1}{2} + y, \frac{1}{2} - x, \frac{1}{2}; \frac{1}{2} - y, \frac{1}{2} + x, \frac{1}{2}.$
---	----------	----------	---	---

8	<i>h</i>	2	$0, \frac{1}{2}, z; 0, \frac{1}{2}, \bar{z}; 0, \frac{1}{2}, \frac{1}{2} + z; 0, \frac{1}{2}, \frac{1}{2} - z;$	$\frac{1}{2}, 0, z; \frac{1}{2}, 0, \bar{z}; \frac{1}{2}, 0, \frac{1}{2} + z; \frac{1}{2}, 0, \frac{1}{2} - z.$
---	----------	---	---	---

hkl : $h+k=2n; l=2n$

4	<i>g</i>	<i>mm</i>	$x, \bar{x}, 0; \bar{x}, x, 0; \frac{1}{2} + x, \frac{1}{2} + x, \frac{1}{2}; \frac{1}{2} - x, \frac{1}{2} - x, \frac{1}{2}.$	
---	----------	-----------	---	--

4	<i>f</i>	<i>mm</i>	$x, x, 0; \bar{x}, \bar{x}, 0; \frac{1}{2} + x, \frac{1}{2} - x, \frac{1}{2}; \frac{1}{2} - x, \frac{1}{2} + x, \frac{1}{2}.$	
---	----------	-----------	---	--

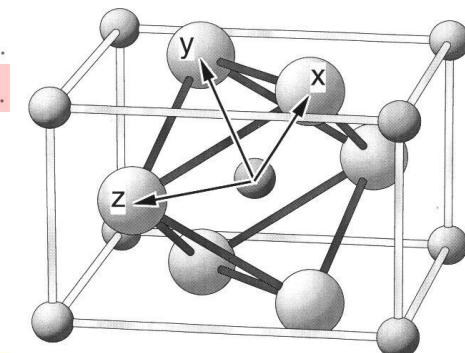
4	<i>e</i>	<i>mm</i>	$0, 0, z; 0, 0, \bar{z}; \frac{1}{2}, \frac{1}{2}, \frac{1}{2} + z; \frac{1}{2}, \frac{1}{2}, \frac{1}{2} - z.$	
---	----------	-----------	---	--

4	<i>d</i>	4	$0, \frac{1}{2}, \frac{1}{4}; \frac{1}{2}, 0, \frac{1}{4}; 0, \frac{1}{2}, \frac{3}{4}; \frac{1}{2}, 0, \frac{3}{4}.$	
---	----------	---	---	--

4	<i>c</i>	2/m	$0, \frac{1}{2}, 0; \frac{1}{2}, 0, 0; 0, \frac{1}{2}, \frac{1}{2}; \frac{1}{2}, 0, \frac{1}{2}.$	
---	----------	-----	---	--

2	<i>b</i>	mmm	$0, 0, \frac{1}{2}; \frac{1}{2}, \frac{1}{2}, 0.$	
---	----------	-----	---	--

2	<i>a</i>	mmm	$0, 0, 0; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}.$	
---	----------	-----	---	--



Structure generator

- **Specify:**

- *Number of nonequivalent atoms*
- *lattice type (P, F, B, H, CXY, CXZ, CYZ) or spacegroup symbol*
 - if existing, you must use a **SG-setting** with inversion symmetry:
 - Si: $\pm(1/8, 1/8, 1/8)$, not $(0,0,0)+(1/4, 1/4, 1/4)$!
- *lattice parameters a, b, c (in Å or bohr)*
- *name of atoms (Si) and fractional coordinates (position)*
 - as numbers (0.123); fractions (1/3); simple expressions ($x-1/2, \dots$)
 - in fcc (bcc) specify just one atom, not the others in $(1/2, 1/2, 0; \dots)$

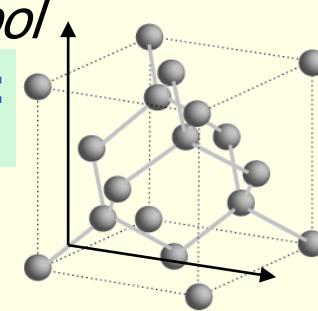
- „**save structure**“

- *updates automatically Z , r_0 , equivalent positions*

- „**set RMT and continue**“: (specify proper “reduction” of NN-distances)

- *non-overlapping „as large as possible“ (saves time, may require $L^{vns}=6(8)$)*
- *RMT for sp (d) - elements 10-20 % smaller than for d (f) elements*
- *largest spheres not more than 50 % larger than smallest sphere*
- *Exception: H in C-H or O-H bonds: RMT~0.6 bohr (RKMAX~3-4)*
- *Do not change RMT in a „series“ of calculations, RMT equal for same atoms*

- „**save structure – save+cleanup**“



Program structure of WIEN2k

■ init_lapw

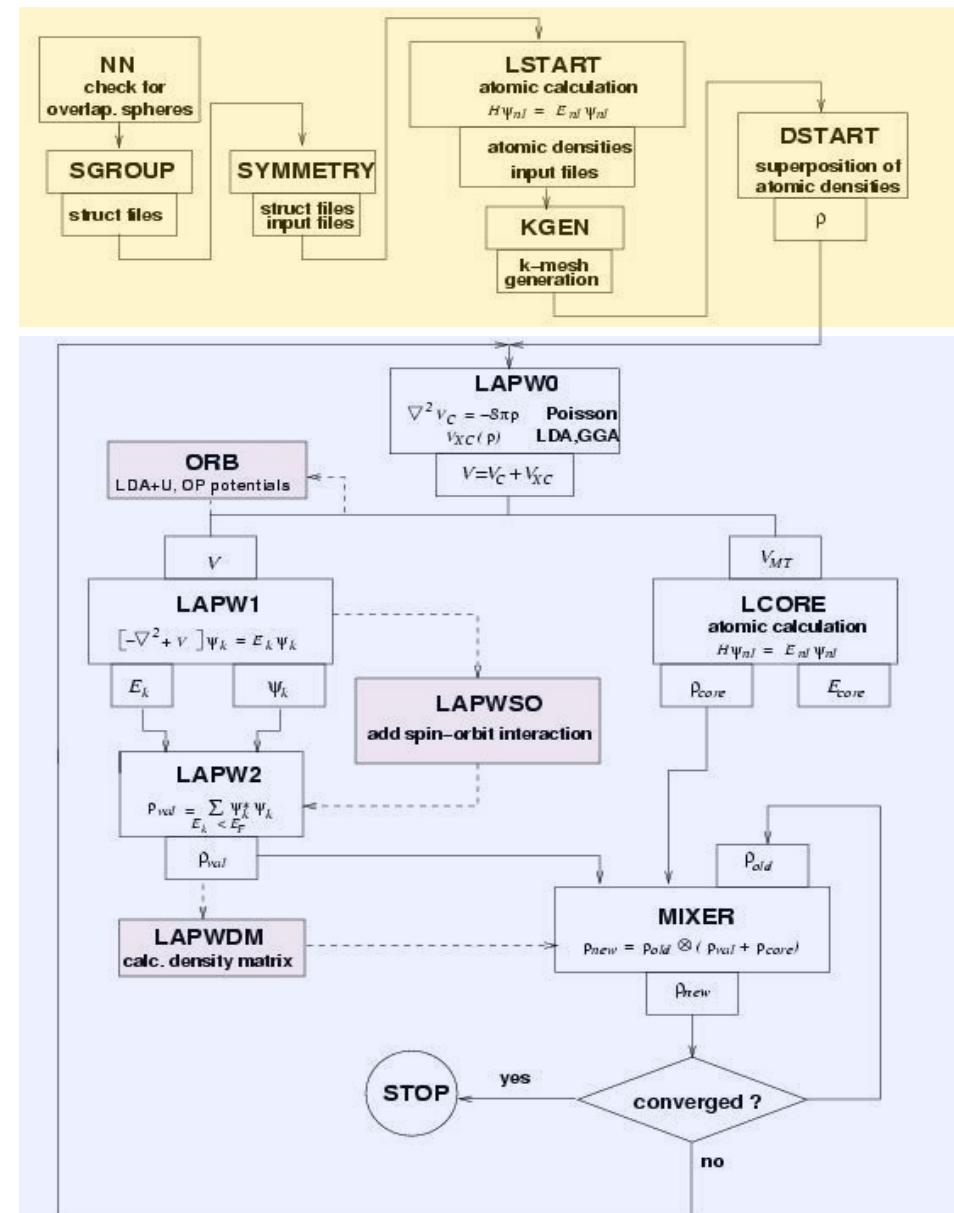
- step-by-step or batch initialization
- symmetry detection (F , I , C -centering, inversion)
- input generation with recommended defaults
- quality (and computing time) depends mainly on **k-mesh** and **R.Kmax** (determines #PW)

■ run_lapw

- scf-cycle
- optional with SO and/or LDA+U
- different convergence criteria (energy, charge, forces)

■ save_lapw tic_gga_100k_rk7_vol0

- cp case.struct and clmsum files,
- mv case.scf file
- rm case.broyd* files



- The convergence criterion in APW is the product of $R_{MT} \cdot Kmax$

$$\Psi = \sum_{K_n}^{KMAX} c_{K_n} e^{iK_n r}$$

- http://www.wien2k.at/reg_user/faq/rkmax.html
- medium quality convergence for **smallest** atom:

- basis set scales with $RKmax^3$
- cputime scales with N_{PW}^3
- increasing Rkmax by 10 %
→ doubles cputime

Rkmax	Element
3.0	H
4.5	Li
5.0	Be, B, Si
5.5	C, P
6.0	N, S
6.5	O, Cl, Na, K, Rb, Cs, Mg, Ca, Sr, Ba, Al
7.0	F
7.5	Sc-Cr, Ga-Br, Y-Mo
8.0	Mn-Zn, Ru-Cd, In-I, La, Ce, Hf-Re
8.5	Os-At, Pr-Lu, Ac-Lr

START with **SMALL** Rkmax (relaxation), **increase/test** later

BZ integration, “FERMI”-methods

- Replace the “integral” of the BZ by a finite summation on a mesh of “k-points”

$$\rho(r) = \sum_n^{E_n < E_F} \int \psi_{k,n}^* \psi_{k,n} d^3k = \sum_{k,n} w_{k,n} \psi_k^* \psi_k$$

- weights $w_{k,n}$ depend on k and bandindex n (occupation)

- for full “bands” the weight is given by “symmetry”

- $w(\Gamma)=1, w(x)=2, w(\Delta)=4, w(k)=8$

- → shifted “Monkhorst-Pack” mesh

- for partially filled bands (metals) one must find the Fermi-energy (integration up to NE) and determine the weights for each state $E_{k,n}$

- linear tetrahedron method+“Bloechl” corrections (TETRA), $T=0$, inconsistent forces

- “broadening methods”

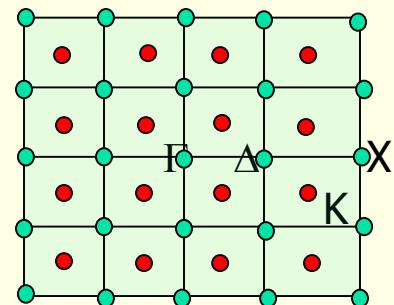
- gauss-broadening (GAUSS 0.005)

- temperature broadening (Fermi-Dirac smearing)

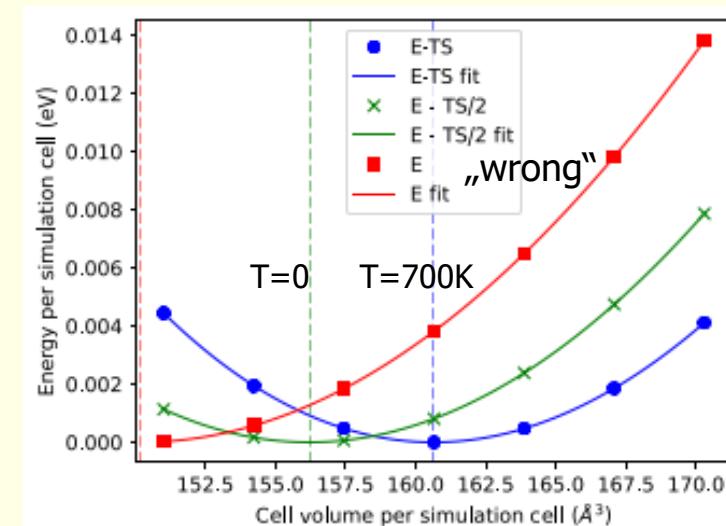
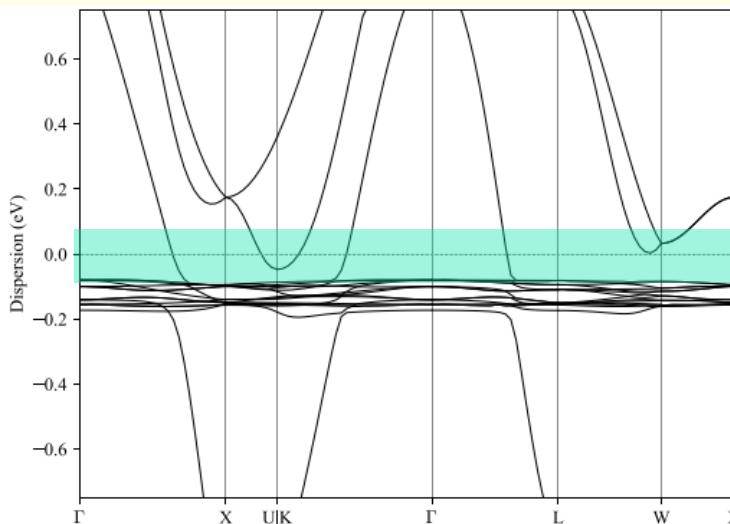
- TEMP 0.002 (-TS/2 correction, extrapolating to $T=0$, inconsistent forces)

- TEMPS 0.002 (finite T ($\sim 300K$) result, -TS corr., **consistent** forces)

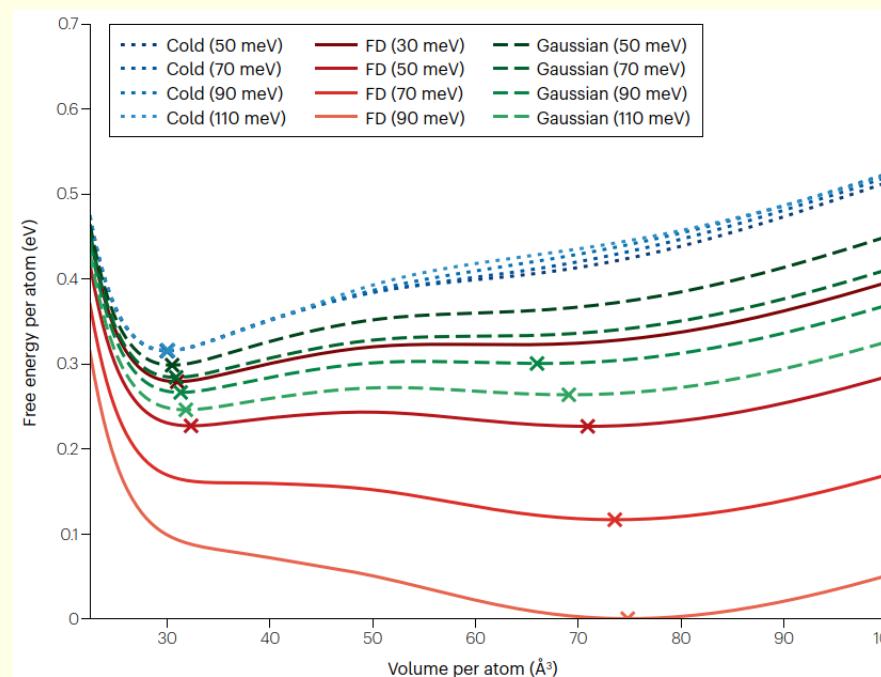
- broadening useful to damp scf oszillations, but dangerous (magnetic moment)



Er in diamond structure



$T=0.0045 \text{ Ry}$
 $\sim 700 \text{ K}$



volume may change by a factor of 2.

- **x kgen** (generates k-mesh and reduces to irreducible wedge using symmetry)
 - *automatically "adds inversion"*
 - time inversion holds and $E(k) = E(-k)$
 - except in magnetic spin-orbit calculations (**x -so kgen**; uses case.ksym file)
 - **x -fbz kgen** (generates „full mesh“ in BZ)
 - *always "shift" the mesh for scf-cycle*
 - gaps often at Γ ! (might not be in your mesh)
 - *small unit cells* and *metals* require large k-mesh (1000-100000)
 - *large unit cells* and *insulators* need only 1-10 k-points
 - use at first a fairly **coarse** mesh for scf/relaxations
 - continue **later** with **finer** mesh
 - mesh was good if nothing changes and scf terminates after few (3) iterations
 - use even finer meshes for DOS, spectra, optics,...

- You can specify the k-mesh by:
 - Total number of k-points in full BZ (typically 1 - 50000)
 - 1000 = 10x10x10 in cubic sym
 - = 13x13x5 in tetragonal with c/a=2.2
 - 3 divisions for the 3 rec.latt.vectors (no check, give correct ratio):
 - 0
 - 4 4 4 (for a cubic insulator), or 20 20 1 (for a metallic surface slab)
 - delta K (in bohr $^{-1}$, typically between 0.5 – 0.02)
 - -1
 - 0.1
- **init_lapw –prec 0-3** (for metals) or **0n-3n** (for insulators) will set the k-mesh automatically according to the size of the cell.
- It will also set **ALL** other parameters (RKMAX, GMAX, L-VNS, RMT, HDLOs, FFT-mesh) according to the desired precision.

- All programs are executed via the „master“ shell-script `x_lapw`
`x lapw2 -up -orb`

- This generates a „def“ file: `lapw2.def`

```
5, 'tin.in2c',          'old',      'formatted'  
6, 'tin.output2up',    'unknown',   'formatted'  
8, 'tin.clmvalup',    'unknown',   'formatted'  
10, './tin.vectorup', 'unknown',   'unformatted'
```

- and executes: `lapw2c lapw2.def`

- All WIEN2k-shell scripts have long and short names:

- `x_lapw; runsp_lapw, runfsm_lapw` → `x; runsp; runfsm`

- All scripts have a „help“ switch „`-h`“, which explains flags and options (without actually execution)

`x -h` `x lapw1 -h`

scf-cycle

■ **run_lapw [options]** (for nonmagnetic cases)

■ <i>-ec 0.0001</i>	<i>convergence of total energy (Ry)</i>
■ <i>-cc 0.0001</i>	<i>convergence of charge distance (e^-)</i>
■ <i>-fc 1.0</i>	<i>convergence of forces (mRy/bohr)</i>
■ <i>-it (-it1,-it2, -noHinv)</i>	<i>iterative diagonalization (large speedup)</i>
■ <i>-p</i>	<i>parallel calculation (needs .machines file)</i>
■ <i>-SO</i>	<i>add spin-orbit (only after „initso“)</i>
■ <i>Spacegroups without inversion use automatically lapw1c, lapw2c (case.in1c,in2c)</i>	

■ **case.scf: master output file, contains history of the scf-cycle**

- *most information is stored with some „labels“ (grep :label case.scf)*

■ :ENE	:DIS	:FER	:GAP	:CTO001	:NTO001	:QTL001
■ :FGL002:	2.ATOM		13.767	13.767	0.000	total forces
■ :LAT	:VOL		:POSxxx			

- ***_lapw -h** „help switch“ of all WIEN2k-scripts
- **help_lapw:**
 - *opens usersguide.pdf; Use ^f keyword to search for an item („index“)*
- **html-version of the UG:** (\$WIENROOT/SRC_usersguide/usersguide.html)
- **http://www.wien2k.at/reg_user**
 - *FAQ page with answers to common questions*
 - *Update information: When you think the program has an error, please check newest version*
 - *Textbook section: DFT and the family of LAPW methods by S.Cottenier*
 - *Mailing-list:*
 - **subscribe** to the list (always use the same email)
 - **full text search** of the „digest“ (your questions may have been answered before)
 - **posting questions:** **Provide sufficient information**, locate your problem (case.dayfile, *.error, case.scf, case.outputX).
 - **„My calculation crashed. Please help.“** This will most likely not be answered.

- „QTL-B“ value too large - STOP (or :WARN): “ghostbands”
 - identify for which **eigenvalue**, **atom** and ℓ it happens, check E_F (`case.scf2`, `case.output2`)
 - identify the corresponding linearization energies in `case.scf1`
 - change the corresponding linearization energy in `case.in1`
 - compare and check with :EPL and :EPH lines in `case.scf2`
 - default E-parameters are adapted automatically but may need changes for
 - surfaces, molecules (negative EF) or heavy elements (EF often larger than 1.0)
 - add a local orbital with \ddot{u} (HDLO) (or adjust its energy)
 - if QTL-B occurs for an atom with large RMT (and you have other atoms with small RMT), reduce RMT („numerical linear dependency“)
- scf-cycle diverges (grep :DIS `case.scf`):
 - check structure (most likely a wrong structure caused divergence);
 - check E-parameters (see above), check :NEC01 (correct number of e^-)
 - `rm *.broyd* case.scf; x dstart`
 - set STIFFER in `case.inm`

- WFFIL

EF=0.634

- 7.00

10 4

(WFPRI, SUPWF)

- 0.30

5 0

(R-MT*K-MAX); MAX L IN WF, V-NMT
global E-param with N other, napw

- 0

0.30

0.000 CONT 1

Es

- 0

-3.72

0.005 STOP 1

Es-LO with search

- 1

-2.07

0.010 CONT 1

Ep with search

- 1

0.30

0.000 CONT 1

Ep-LO

- 2

0.30

0.010 CONT 1

0/1/2...LAPW/APW+lo/HLO

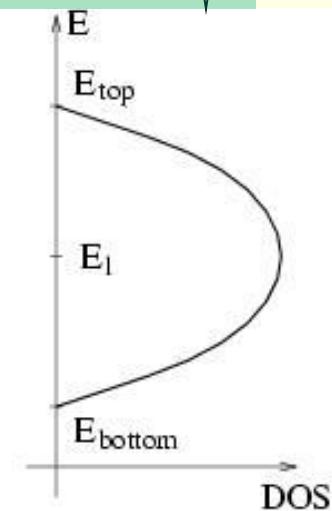
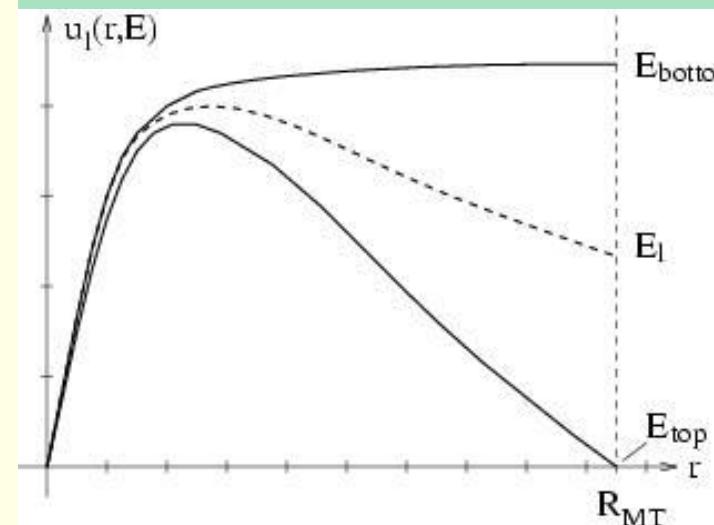
- K-VECTORS FROM UNIT:4

-7.0 1.5 16 emin/emax; nband

$$\Psi = \sum_{K_n}^{KMAX} c_{K_n} e^{iK_n r}$$

$$\Phi_{K_n} = \sum_l^{l \text{ max}} A_{lm} u_l(E_l, r) Y_{lm}$$

$$H_{n,m}^{NS} = \langle \Phi_l | V_{LM}^{NS} | \Phi_{l'} \rangle$$



- **f (d) wavefunctions have a large E-dependency in cases with large RMT**
- For **high precision** calculations extend the basis set with a **HDLO** (high derivative LO):

$$\Phi_{K_n} = \sum_l A_{lm}(K_n) u_l(E_l, r) Y_{lm}$$

APW

$$\phi_{l,atom} = (A_{lm} u_{lm} + B_{lm} \dot{u}_l) Y_{lm}$$

lo

$$\phi_{l,atom} = (A_{lm} u_{lm} + C_{lm} \ddot{u}_l) Y_{lm}$$

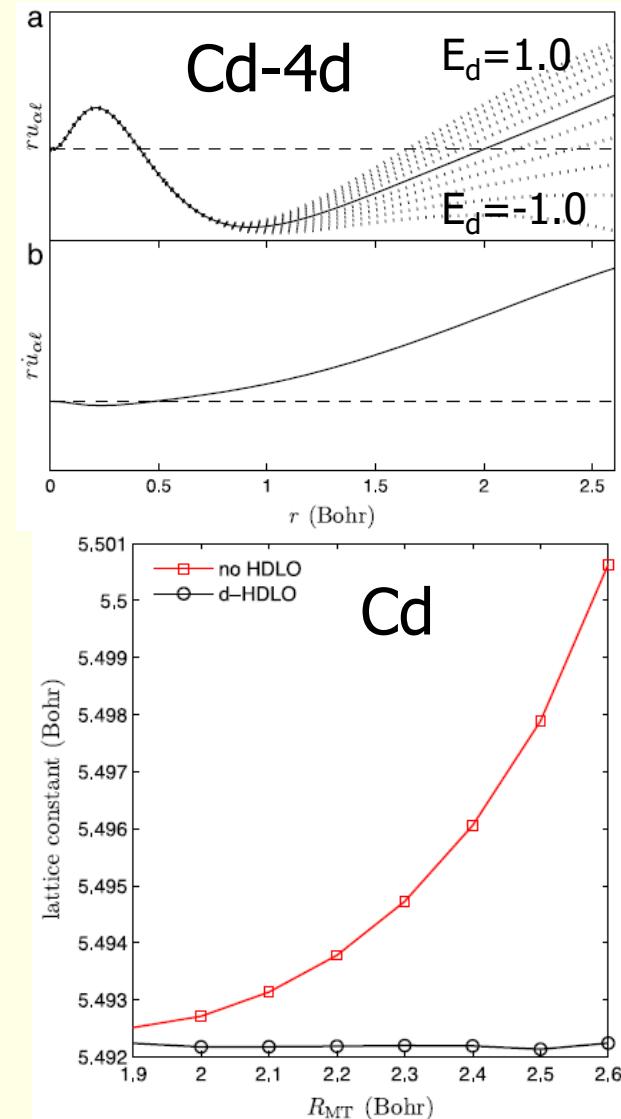
HDLO

- 2 0.30 0.010 CONT 1
- 2 0.30 0.010 CONT 2

APW+lo

HDLO

- F.Karsai et al., CPC 220, 230 (2017)



case.klist, case.in2

- ```
■ GAMMA 0 0 0 40 1.0 IX, IY, IZ, IDIV, WEIGHT
■ 1 0 0 40 6.0
■ ...
■ X 40 0 0 40 3.0
■ END
```

case.in2:

- TOT (TOT, FOR, QTL, EFG, FERMI)  
■ -9.0 16.0 0.50 0.05 EMIN, NE, ESEPARMIN, ESEPARO
  - TETRA 0.000 (GAUSS, ROOT, TEMP, TETRA, ALL eval)  
■ 0 0 4 0 4 4 6 0 6 4  
■ 0 0 4 0 4 4 6 0 6 4
  - 14. GMAX (automatically larger for small spheres H)
  - FILE FILE/NOFILE write reciprlist

$$\rho(r) = \sum_{LM} \rho_{LM}(r) Y_{LM}(\hat{r})$$

$$\rho(r) = \sum_G \rho_G e^{iGr}$$

- Energy bands
  - *classification of irreducible representations*
  - *'character-plot' (emphasize a certain band-character)*
- Density of states
  - *including partial DOS with l and m- character (eg.  $p_x, p_y, p_z$ )*
- Electron density, potential
  - *total-, valence-, difference-, spin-densities,  $\rho$  of selected states*
  - *1-D, 2D- and 3D-plots (Xcrysden)*
  - *X-ray structure factors*
  - *Bader's atom-in-molecule analysis, critical-points, atomic basins and charges ( $\nabla \rho \cdot \vec{n} = 0$ )*
  - *spin+orbital magnetic moments (spin-orbit / LDA+U)*
- Hyperfine parameters
  - *hyperfine fields (contact + dipolar + orbital contribution)*
  - *Isomer shift*
  - *Electric field gradients*
  - *NMR chemical and Knight shifts*

- be sure to have `case.vector` on a dense tetrahedral mesh after a scf calculation

- *eventually:*

- `x kgen`
- edit `case.in1` (larger Emax)
- `x lapw1`

- **`x lapw2 -qtl`**

$$\Psi_n * \Psi_n = 1 = q_{out} + \sum_t^{at} \sum_l q_{t,l}$$

- **`case.outputt`**

- *integrated DOS*

- **`case.dos1ev (3ev)`**

- *text-file for plotting*
- *E-zero at  $E_F$*

Session: **TiC**  
 /susi/pbla/ha/lapw/TiC

## Density of states

- `x lapw2 -qtl` Calculate partial charges  interactively
- `edit TiC.int` Edit input-file for TETRA
- `x tetra` Calculate partial DOS  interactively
- `edit TiC.outputt` Check output of TETRA
- `dosplot` Plot DOS

Session: **TiC**  
 /susi/pbla/ha/lapw/TiC/TiC.int

### File:

/susi/pbla/ha/lapw/TiC/TiC.int

[continue with DOS](#)

[Save](#)

Download this file: [\[icon\]](#)

### Header from TiC.qtl:

```
ATOM 1 tot,0,1,2,3,xdos(i,j),j=1,i),i=1,1xdos2)
ATOM 2 tot,0,1,2,D-eg,D-t2g,3
```

### Title

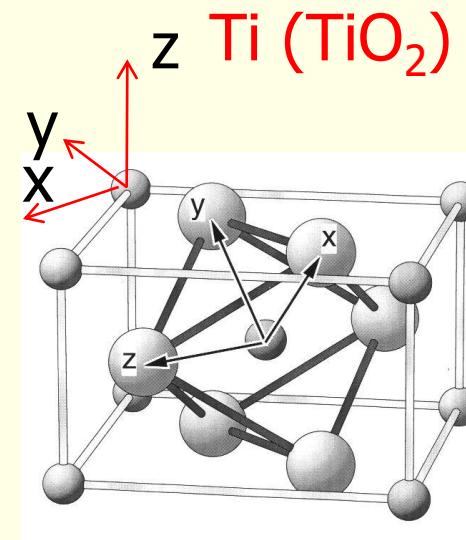
|       |       |          |       |                                         |
|-------|-------|----------|-------|-----------------------------------------|
| -0.50 | 0.002 | 1.500    | 0.003 | EMIN, DE, EMAX, Gauss-broadening (>;de) |
| 3     |       |          |       | NUMBER OF DOS-CASES specified below     |
| 0     | 1     | total    |       | atom, case=column in qtl-header, label  |
| 1     | 2     | Atom1-s  |       |                                         |
| 2     | 5     | Atom2-eg |       |                                         |

# partial charges:

- local rotation matrix:

- transfers  $z$  ( $y$ ) into highest symmetry
- reduces terms in LM series
- "chemical" interpretation
  - $p_x$  is different from  $p_y$

$$\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



- see *case.struct* and *case.outputs*

- **x qtl** (instead of `x lapw2 -qtl`)

- **f-orbitals**
- **qtls for different coordinate system** (eg. "octahedral" in  $TiO_2$ )
- **relativistic basis ( $p_{1/2}$ - $p_{3/2}$  or  $d_{3/2}$ - $d_{5/2}$  splitting in so calculation)**
- **for angular dependend TELNES (ISPLIT 88, 99)**

## ■ Energy bands

- *classification of irreducible representations*
- *'character-plot' (emphasize a certain band-character)*

## ■ Density of states

- *including partial DOS with l and m- character (eg.  $p_x, p_y, p_z$ )*

## ■ Electron density, potential

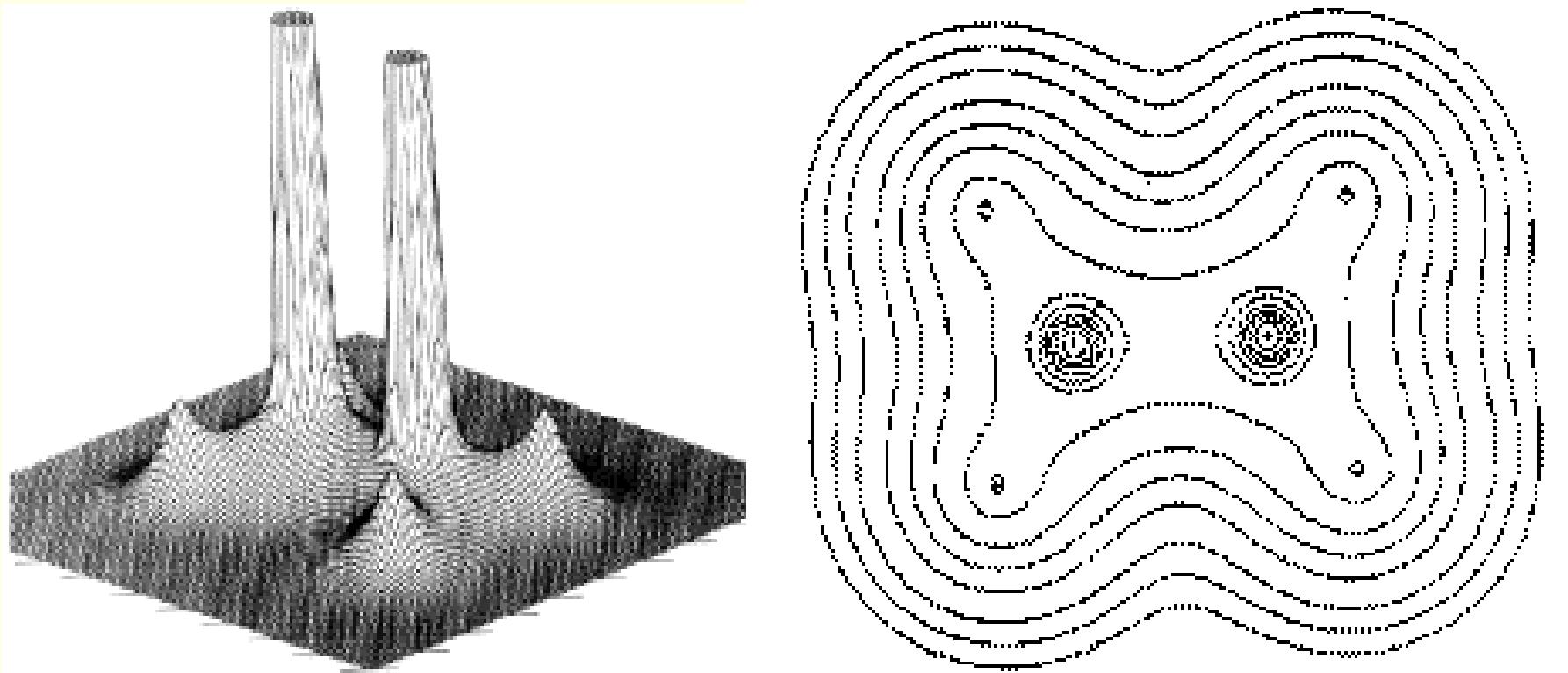
- *total-, valence-, difference-, spin-densities,  $\rho$  of selected states*
- *1-D, 2D- and 3D-plots (Xcrysden)*
- *X-ray structure factors*
- *Bader's atom-in-molecule analysis, critical-points, atomic basins and charges  
 $(\nabla \rho \cdot \vec{n} = 0)$*
- *spin+orbital magnetic moments (spin-orbit / LDA+U)*

## ■ Hyperfine parameters

- *hyperfine fields (contact + dipolar + orbital contribution)*
- *Isomer shift*
- *Electric field gradients*

- Theory to characterize atoms and chemical bonds from the topology of the electron density, by R.F.Bader  
([http://www.chemistry.mcmaster.ca/faculty/bader/aim/aim\\_0.html](http://www.chemistry.mcmaster.ca/faculty/bader/aim/aim_0.html))

Electron density of  $\text{C}_2\text{H}_4$

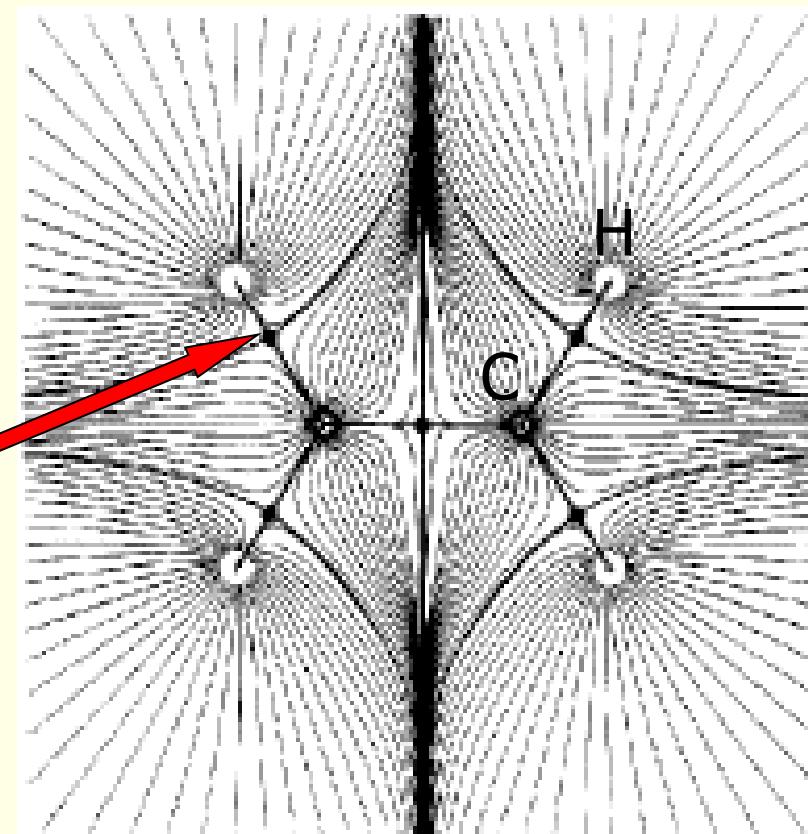


■ Bonds are characterized by „critical points“, where  $\nabla \rho = 0$

- density maximum: (3,-3); 3 negative curvatures  $\lambda$ , (at nucleus or non-NM)
- bond CP: (3,-1): 2 negative, 1 positive  $\lambda$  (saddle point)
  - positive (and large) Laplacian: ionic bond
  - negative Laplacian: covalent bond
- bridge CP: (3,1)
- cage CP: (3,3) (minimum)

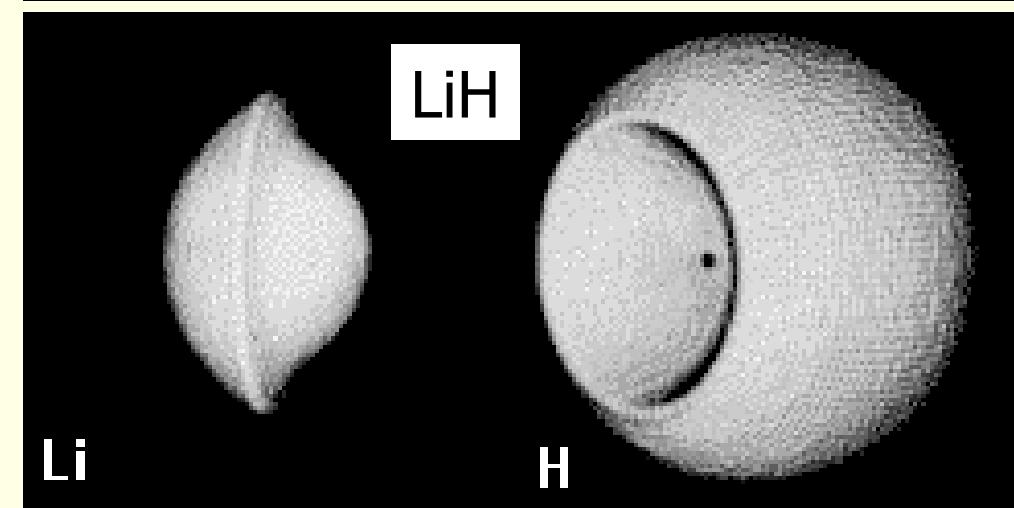
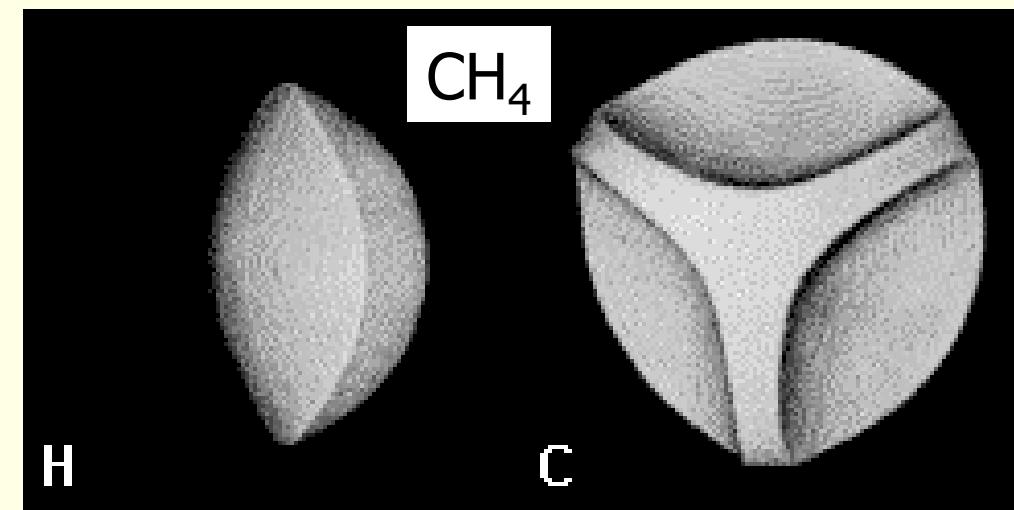
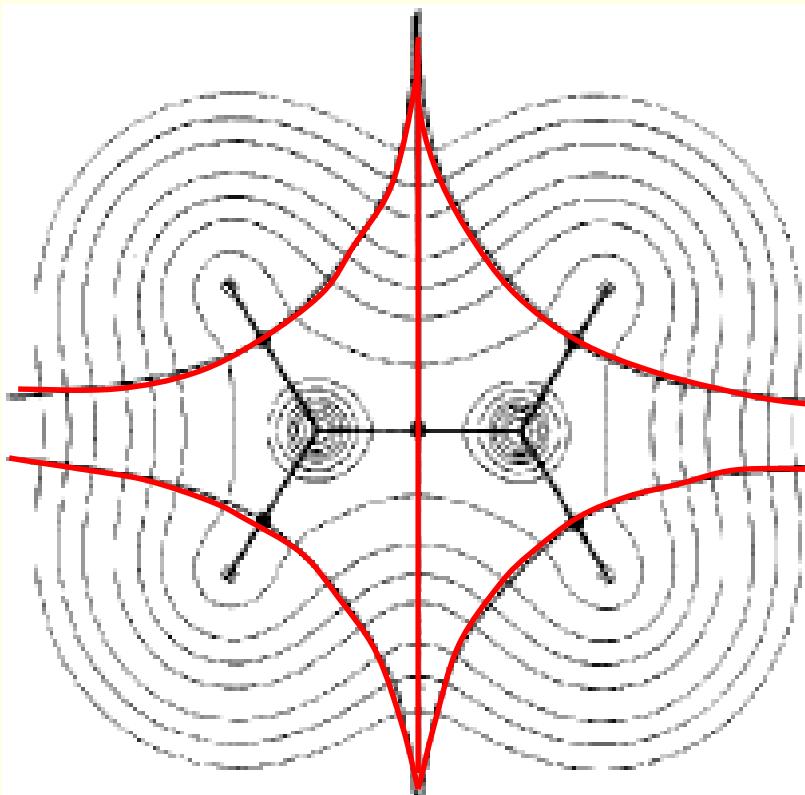
(3,-1) BCP

trajectories of constant  $\nabla \rho$   
originating at CPs in  $C_2H_4$



- “Atoms” are regions within a zero-flux surface  $\vec{\nabla} \rho \cdot \vec{n} = 0$

$\rho$  of  $C_2H_4$  with zero-flux lines defining atomic basins



- 2 tools: **x aim** or **critic2** (see unsupported software)
- Bader analysis of some inorganic compounds:

|                 | $\rho(\text{e}/\text{A}^3)$ | $\Delta\rho(\text{e}/\text{A}^5)$ | Q (e) |
|-----------------|-----------------------------|-----------------------------------|-------|
| Cl <sub>2</sub> | 1.12                        | -6.1                              | -     |
| I <sub>2</sub>  | 0.48                        | -0.9                              | -     |
| TiC             | 0.51                        | 1.8                               | 1.7   |
| TiN             | 0.47                        | 3.9                               | 1.7   |
| TiO             | 0.43                        | 5.8                               | 1.5   |
| KCl             | 0.08                        | 1.2                               | 0.6   |

Cl<sub>2</sub> more covalent than I<sub>2</sub>

more ionic, but less charge?

less ionic than TiC ?

- **x aim –up:** gives “unique” magnetic moments/atom

## x aim

- You must have a “good” scf-density (case.clmsum)
  - no core leakage, LMs up to  $L=8-10$  in case.in2

## SURF

```
1 atom in center of surface (including MULT)
20 0.0 1.570796327 theta, 20 points, from zero to pi/2
20 0.0 0.785398163 phi, from 0 to pi/4 (depends on symmetry!!)
0.07 1.0 4 step along gradient line, rmin (has reached an atom)
1.65 0.1 initial R for search, step (a.u)
3 3 3 nshell
IRHO "INTEGRATE" rho
WEIT WEIT (surface weights are available in case.surf)
30 30 radial points outside min(RMIN,RMT)
END
```

---

## CRIT

```
1 atom around you search for critical points
ALL two, three, four, all (dimers, trimers,...all=2+3)
3 3 3 nshell
END
```

extractaim\_lapw: → critical\_points\_ang (converted units)  
:PC x, y, z,  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ , ch, laplacian, rho

## ■ Total energy and forces

- *optimization of internal coordinates, (MD, BROYDEN)*
- *cell parameter only via  $E_{tot}$  (no stress tensor)*
- *elastic constants for cubic, hexagonal, and tetragonal cells*
- *Phonons via supercells*
  - interface to PHONON (K.Parlinski) – bands, DOS, thermodynamics, neutrons
  - interface to PHONOPY (A. Togo)
    - [http://www.wien2k.at/reg\\_user/unsupported](http://www.wien2k.at/reg_user/unsupported)

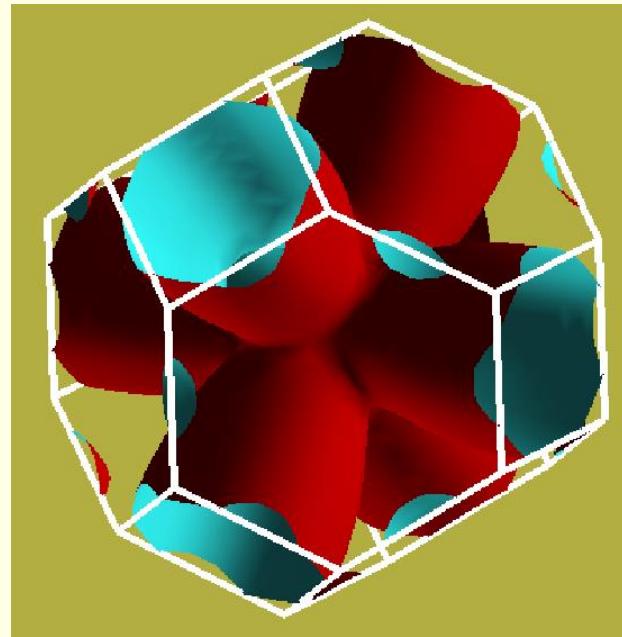
## ■ Spectroscopy

- *core level shifts*
- *X-ray emission, absorption, electron-energy-loss (with core holes)*
  - core-valence/conduction bands including matrix elements and angular dep.
- *optical properties (dielectric function in RPA approximation, JDOS including momentum matrix elements and Kramers-Kronig)*

## ■ fermi surface: 2D, 3D (using XcrysDen)

- **`xcrysden --wien_fermisurface tin.struct`**

- choose a good k-mesh (eg. 10000 points)
- plot the FS for all bands which cross  $E_F$  and compare to band structure



- for 2D plots there is also a WIEN2k-tool „fsgen“ (see UG)
- SKEAF ([www.wien2k.at/reg\\_users/unsupported](http://www.wien2k.at/reg_users/unsupported)): quantum oszillations

## ■ Total energy and forces

- *optimization of internal coordinates, (MD, BROYDEN)*
- *cell parameter only via  $E_{tot}$  (no stress tensor)*
- *elastic constants for cubic, hexagonal, and tetragonal cells*
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## ■ Spectroscopy

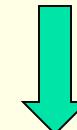
- *core level shifts*
- *X-ray emission, absorption, electron-energy-loss (with core holes)*
  - core-valence/conduction bands including matrix elements and angular dep.
- *optical properties (dielectric function in RPA approximation, JDOS including momentum matrix elements and Kramers-Kronig)*

## ■ fermi surface: 2D, 3D (using XcrysDen)

$$E_{A_xB_y}^{cohes.} = E^{crystal} - xE_A^{atom} - yE_B^{atom}$$

- $E^{crystal}$ : scalar-relativistic valence (or approx. SO)
- $E^{atom}$  : LSTART: fully-relativistic → inconsistent description
  - for heavier elements (2<sup>nd</sup> row): supercell with one atom in a ~30-40 bohr **distorted** FCC box (identical RMT, 1 k-point, **equivalent** RKmax, spinpolarized, iterative diagonalization)

$RMT^A=2.0, RMT^B=1.8, Rkmax=7 \rightarrow RKmax^A=7*2.0/1.8, RKmax^B=7$



## ■ Lattice parameters, volume, c/a ratio only via total energies:

- *x optimize [-job "run -ec 0.000001" -save pbe]*: creates a series of "struct" files + script "optimize.job"
    - select volume or c/a, ...
    - select number of cases and desired changes in volume (in % of  $V_0$ )
  - *edit optimize.job*
    - adapt to your need: change / uncomment various lines, eg.:
      - replace "run" by "runsp" or add options (-min -fc 1 -orb -i 80 -p -fc 1.0)
      - modify "save\_lapw" line (with more specific names)
  - *execute optimize.job*
  - *plot (analyse) the results ( eplot -a pbe )*
- 
- combinations of volume and c/a are possible: *2Doptimize*
    - "x optimize" always uses *case\_initial.struct* (if present)
    - do a "volume" optimization to create *case\_vol\_xx.struct* files
    - copy the respective *case\_vol\_xx.struct* file to *case\_initial.struct*
    - x optimize with "c/a" for this particular volume and proceed as above.
  - *optimize\_abc\_lapw*: for 2D (*a,c*) or 3D (*a,b,c*) optimization

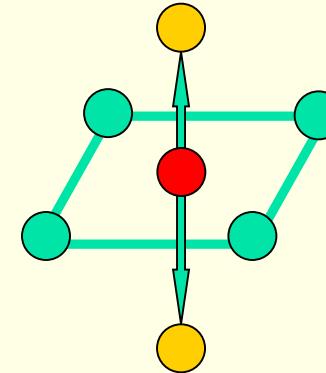
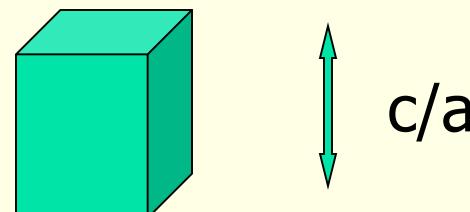
## ■ WIEN „preserves“ symmetry:

### ■ *c/a optimization of „cubic“ TiC:*

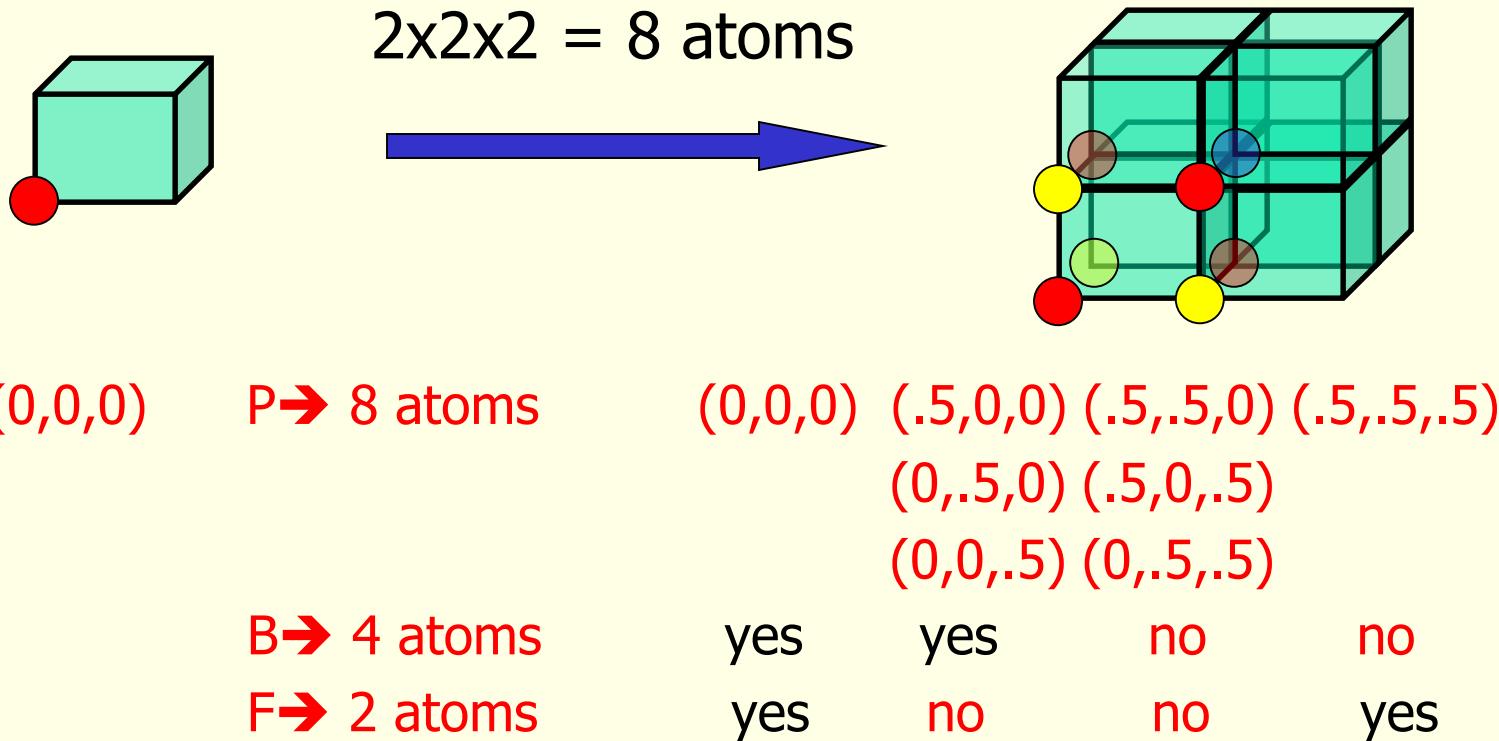
- change c lattice parameter in TiC.struct (tetragonal distortion, #sym.op=0)
- init\_lapw
- change c back to cubic
- x optimize ...

### ■ „Jahn-Teller“ distortion:

- when you start with a perfect octahedra, you will never get any distortion
- → start with slightly distorted positions

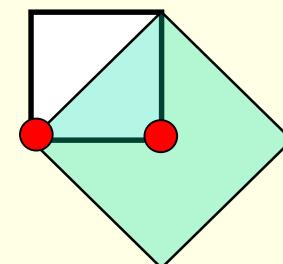


# Supercells (impurities, vacancies, alloys)



4x4x4 supercells: P (64), B (32), F (16) atoms

$\sqrt{2} \times \sqrt{2}$  supercells (1 → 2 atoms)



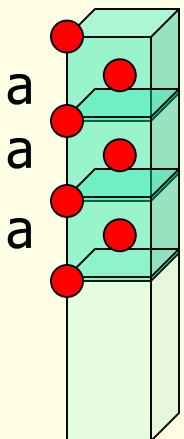
- Program „supercell“:
  - *start with „small“ **struct** file*
  - *specify number of repetitions in x,y,z (only **integers**, e.g. 2x2x1)*
  - *specify **P**, **B** or **F** lattice*
  - *add „vacuum“ for **surface** slabs (only (001) indexed surfaces)*
  - *shift all atoms in cell*
- You must break symmetry !!! (otherwise sgroup will restore your original struct file)
  - *replace (impurities, vacancies) or*
  - *displace (phonons) or*
  - *label at least 1 atom (core-holes, specific magnetic order; change "Fe" to "Fe1"; this tells the **symmetry**-programs that Fe1 is NOT a Fe atom!!)*
- „supercell“ works only along unit-cell axes!!!

- requires octave (matlab) and xcrysden (visualization)
- allows complex operations on struct-files

```
octave
s=loadstruct("GaN.struct")
make an orthorhombic supercell and visualize it
a=[1 0 0; 1 1 0; 0 0 2]
sout=makesupercell (s,a);
showstruct(sout);
save it as test.struct
savestruct (sout,"test.struct");
get help on all commands
helpstruct
```

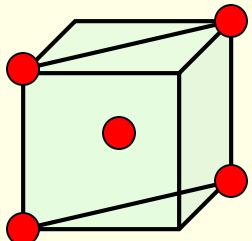
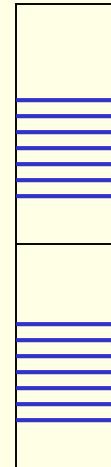
- 2D-slabs with finite number of layers with „vacuum“ in 3<sup>rd</sup> dimension

bcc (001) 7 layers:



$(0 \ 0 \ 6z)$   
 $(.5 \ .5 \ 5z)$   
 $(0 \ 0 \ 4z)$   
 $(.5 \ .5 \ 3z)$   
 $(0 \ 0 \ 2z)$   
 $(.5 \ .5 \ z)$   
 $(0 \ 0 \ 0)$

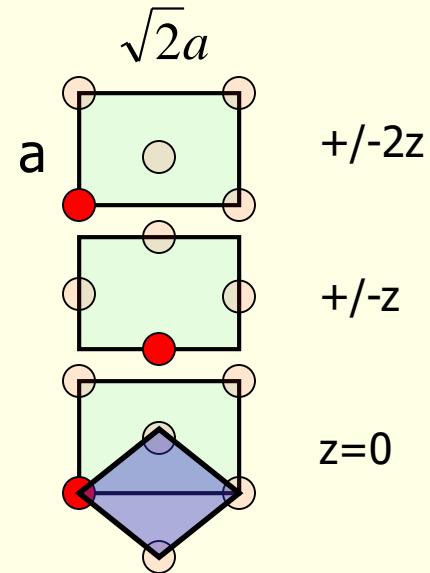
$(.5 \ .5 \ +/-3z)$  with lattice parameters:  
 $(0 \ 0 \ +/-2z)$   $a, a, c = (3a + 15-20 \text{ bohr vacuum})$   
shift to  $(.5 \ .5 \ +/-z)$   
 $\rightarrow$   $(0 \ 0 \ 0)$   $z = a/2c$   
inversion



bcc (110):

orthorhombic CXY-lattice:  $a, \sqrt{2}a, c$

$(0 \ 0 \ 0)$   $z = a/\sqrt{2}a$   $c$   
 $(0 \ .5 \ +/-z)$   
 $(0 \ 0 \ +/-2z)$



# Total energies and atomic forces

(Yu et al.; Kohler et al.)

## ■ Total Energy:

- *Electrostatic energy*
- *Kinetic energy*
- *XC-energy*

$$U[\rho] = \frac{1}{2} \int d^3\vec{r} \rho(\vec{r}) V_{es}(\vec{r}) + \frac{1}{2} \sum_{\alpha} Z_{\alpha} V_{es}^{\alpha}(\vec{r})$$

$$T[\rho] = \sum_i n_i \varepsilon_i - \int d^3\vec{r} \rho(\vec{r}) V_{eff}(\vec{r})$$

$$E_{xc}[\rho] = \int d^3\vec{r} \rho(\vec{r}) \varepsilon_{xc}(\vec{r})$$

## ■ Force on atom $\alpha$ :

$$\vec{F}^{\alpha} = \frac{-dE_{tot}}{d\vec{R}_{\alpha}} = F_{HF}^{\alpha} + F_{core}^{\alpha} + F_{val}^{\alpha}$$

- *Hellmann-Feynman-force*

- *Pulay corrections*

- Core
- Valence

- expensive, contains a summation of matrix elements over all occupied states

$$F_{HF}^{\alpha} = Z_{\alpha} \sum_{m=-1}^1 \lim_{r_{\alpha} \rightarrow 0} \frac{V_{1m}^{es}(r_{\alpha})}{r_{\alpha}} \nabla_{\alpha} [r_{\alpha} Y_{1m}(\hat{r})]$$

$$F_{core}^{\alpha} = - \int \rho_{core}(r) \nabla_{\alpha} V_{eff}(r) d\vec{r}$$

$$F_{val}^{\alpha} = \int_{\alpha} V_{eff}(r) \nabla_{\alpha} \rho_{val}(r) d\vec{r} + \sum_{k,i} n_i \sum_{K,K'} c_i^*(K') c_i(K) \times$$

$$[(K^2 - \varepsilon_i) \oint \phi_{K'}^*(r) \phi_K(r) dS_{\alpha} - i(K - K') \langle \phi_{K'} | H - \varepsilon_i | \phi_K \rangle_{\alpha}]$$

## ■ Forces only for “free” structural parameters:

- $\text{NaCl}$ :  $(0,0,0), (0.5,0.5,0.5)$  : all positions fixed by symmetry
- $\text{TiO}_2$ :  $\text{Ti} (0,0,0)$ ,  $\text{O} (u,u,0)$ : one free parameter ( $u,x,y,z$ )

## ■ Forces are only calculated when using “-fc”:

- `run_lapw -fc 1.0` ( $m\text{Ry}/\text{bohr}$ )

|   |                       |                                                                                       |
|---|-----------------------|---------------------------------------------------------------------------------------|
| ■ | grep :fgl002 case.scf |                                                                                       |
| ■ | 200.                  | partial                                                                               |
| ■ | -130.                 | partial                                                                               |
| ■ | 140.                  | partial                                                                               |
| ■ | 135                   | partial only $F_{\text{HF}} + F_{\text{core}}$                                        |
| ■ | 120                   | partial forces converging                                                             |
| ■ | 122                   | partial $\rightarrow$ changes “TOT” to “FOR” in case.in2                              |
| ■ | 121                   | $F_{\text{HF}} + F_{\text{core}} + F_{\text{val}}$ , only this last number is correct |
| ■ | -12.3                 | <b>total</b>                                                                          |

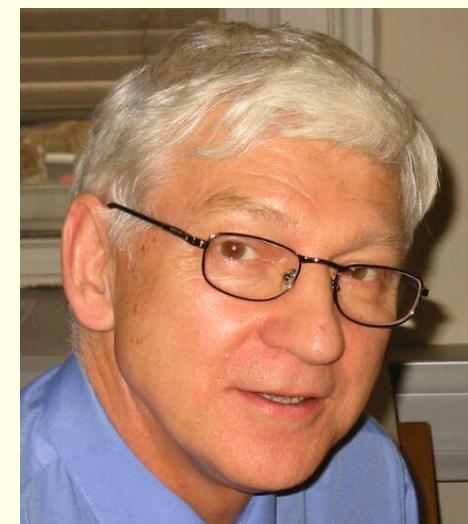
## ■ Forces are useful for

- *structural optimization (of internal atomic positions)*
- *phonons*

# Calculations of Phonons: The Direct Method



WIEN2k + Phonon  
WIEN2k + Phonopy  
WIEN2k + Phon



*Copyright by K.Parlinski*

Phonopy: <https://phonopy.github.io/phonopy/>

Phonon: <https://computingformaterials.com>

Phon: <http://www.homepages.ucl.ac.uk/~ucfbdxu/phon/>

by A.Togo. Most used phonon code, free.

by K.Parlinski. Graphical user interface, expensive

by D.Alfe. free

## THEORY OF DIRECT METHOD

System energy  $E$  (at  $T = 0$ ) as a function of atomic positions  $\mathbf{R}(\mathbf{n}, \mu)$  is

$$E(\mathbf{R}(\mathbf{n}, \mu), \dots, \mathbf{R}(\mathbf{m}, \nu), \dots) = E_o + \frac{1}{2} \sum_{\mathbf{n}, \mu, \mathbf{m}, \nu} \Phi(\mathbf{n}, \mu, \mathbf{m}, \nu) \mathbf{U}(\mathbf{n}, \mu) \mathbf{U}(\mathbf{m}, \nu)$$

where the *force constant matrix* are

$$\Phi_{i,j}(\mathbf{n}, \mu, \mathbf{m}, \nu) = \left. \frac{\partial^2 E}{\partial \mathbf{R}_i(\mathbf{n}, \mu) \partial \mathbf{R}_j(\mathbf{m}, \nu)} \right|_o$$

is defined at  $\left. \frac{\partial E}{\partial \mathbf{R}_i(\mathbf{n}, \mu)} \right|_o = 0$ .

The *dynamical matrix* is defined as

$$\mathbf{D}(\mathbf{k}; \mu, \nu) = \frac{1}{\sqrt{M_\mu M_\nu}} \sum_{\mathbf{m}} \Phi(0, \mu; \mathbf{m}, \nu) \exp\{-2\pi i \mathbf{k} \cdot [\mathbf{R}(0, \mu) - \mathbf{R}(\mathbf{m}, \nu)]\}$$

$\mathbf{m}$  runs over *all* atoms. Diagonalization of the dynamical matrix

$$\omega^2(\mathbf{k}, j) \mathbf{e}(\mathbf{k}, j) = \mathbf{D}(\mathbf{k}) \mathbf{e}(\mathbf{k}, j)$$

gives phonon frequencies  $\omega^2(\mathbf{k}, j)$  and polarization vectors  $\mathbf{e}(\mathbf{k}, j)$ .

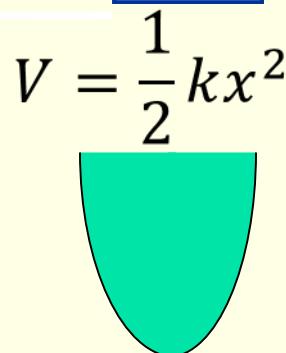
Any atomic displacement  $\mathbf{U}(\mathbf{m}, \nu)$  generates forces

$$\mathbf{F}(\mathbf{n}, \mu) = -\partial E / \partial \mathbf{R}(\mathbf{n}, \mu)$$

on all other atoms. Hence

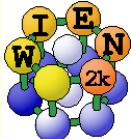
$$F_i(\mathbf{n}, \mu) = -\sum_{\mathbf{m}, \nu, j} \Phi_{i,j}(\mathbf{n}, \mu, \mathbf{m}, \nu) U_j(\mathbf{m}, \nu)$$

Master equation of direct method.



$n, m$ : cells

$\mu, \nu$ : atoms



## CUMMULANT FORCE CONSTANTS

Displace an atom by  $\mathbf{U}(\mathbf{m}, \nu)$

$$F_i(\mathbf{n}, \mu) = - \sum_{\mathbf{L}} \Phi_{i,j}(\mathbf{n}, \mu, \mathbf{m} + \mathbf{L}), \nu) U_j(\mathbf{m}, \nu)$$

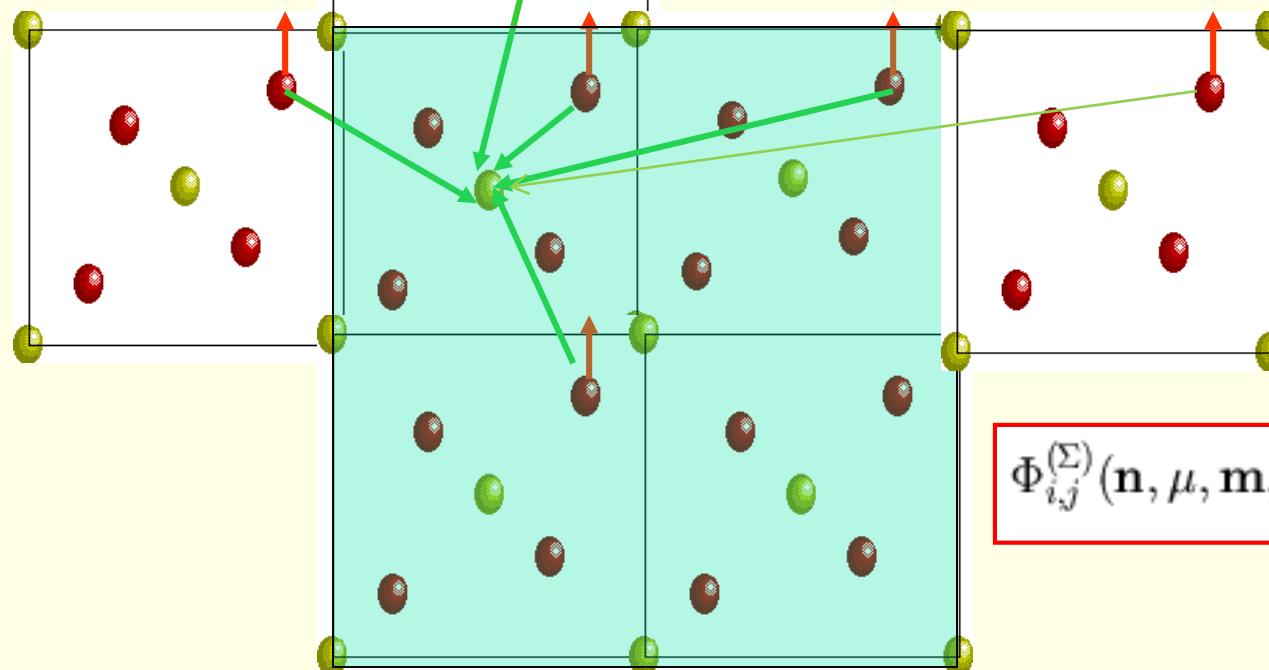
$\mathbf{L} = (L_a, L_b, L_c)$  are the indices of supercell lattice constants.  
or

$$F_i(\mathbf{n}, \mu) = -\Phi_{i,j}^{(\Sigma)}(\mathbf{n}, \mu, \mathbf{m}, \nu) U_j(\mathbf{m}, \nu)$$

where the **cummulant force constant** is

$$\Phi_{i,j}^{(\Sigma)}(\mathbf{n}, \mu, \mathbf{m}, \nu) = \sum_{\mathbf{L}} \Phi_{i,j}(\mathbf{n}, \mu, \mathbf{m} + \mathbf{L}, \nu)$$

$\mathbf{L}$  runs over all supercell images.





# Supercell dynamical matrix. Exact wave vectors.

Conventional dynamical matrix:

$$\mathbf{D}(\mathbf{k}; \mu, \nu) = \frac{1}{\sqrt{M_\mu M_\nu}} \sum_{\mathbf{m}} \Phi(0, \mu; \mathbf{m}, \nu) \exp\{-2\pi i \mathbf{k} \cdot [\mathbf{R}(0, \mu) - \mathbf{R}(\mathbf{m}, \nu)]\}$$

Supercell dynamical matrix:

$$\mathbf{D}^{(SC)}(\mathbf{k}; \mu, \nu) = \frac{1}{\sqrt{M_\mu M_\nu}} \sum_{\mathbf{m} \in SC} \Phi^{(SC)}(0, \mu; \mathbf{m}, \nu) \exp\{-2\pi i \mathbf{k} \cdot [\mathbf{R}(0, \mu) - \mathbf{R}(\mathbf{m}, \nu)]\}$$

These two matrices are equal if

$$\mathbf{D}^{(SC)}(\mathbf{k}; \mu, \nu) = \mathbf{D}(\mathbf{k}; \mu, \nu)$$

- **interaction range** is confined to **interior** of supercell (**supercell is big enough**)
- wave vector is **commensurate with the supercell** and fulfils the condition (independent of interaction range):

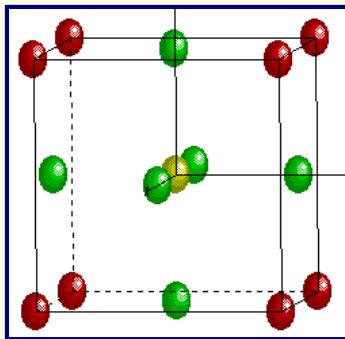
$$\exp\{-2\pi i \mathbf{k}_s \cdot \mathbf{L}\} = 1$$

At wave vectors  $\mathbf{k}_s$  the phonon frequencies are “exact”, provided the **supercell contains the complete list of neighbors**.

Wave vectors  $\mathbf{k}_s$  are commensurate with the supercell size.

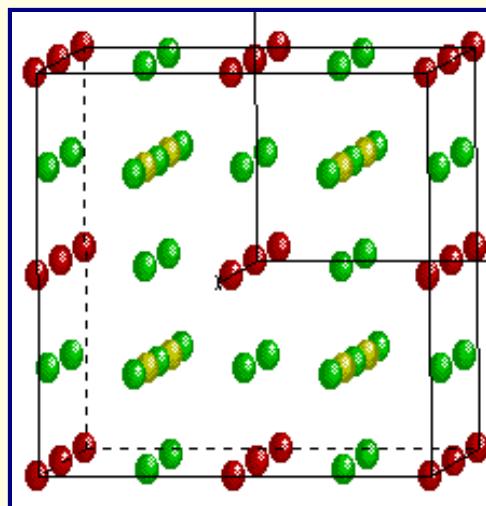
# Exact wave vectors

$1 \times 1 \times 1$



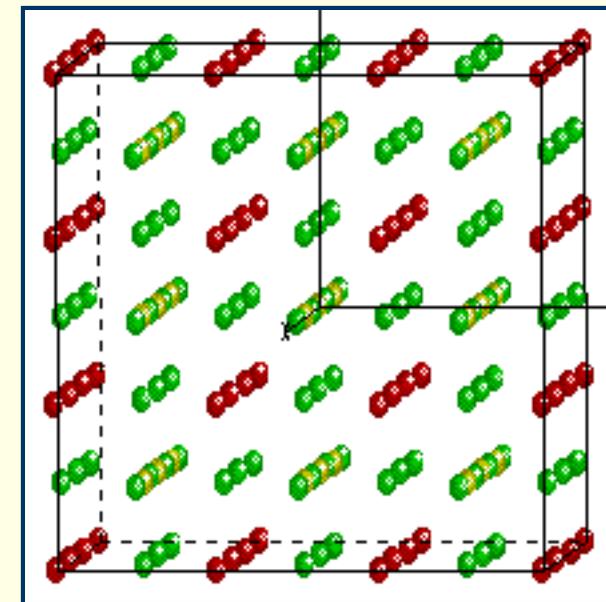
Exact:  $\Gamma$

$2 \times 2 \times 2$



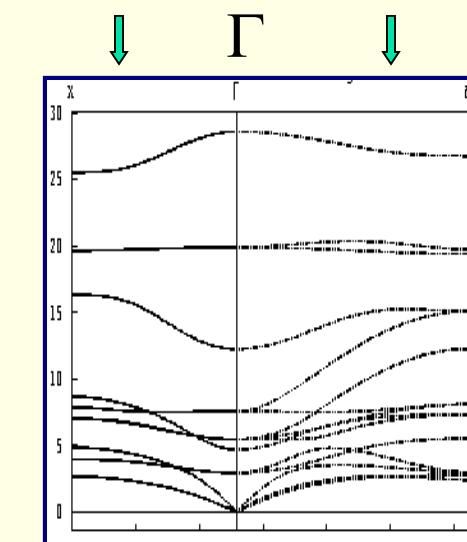
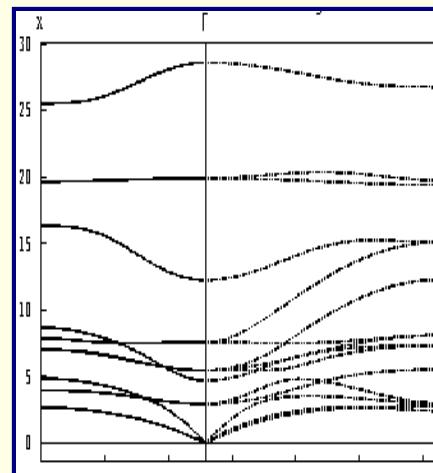
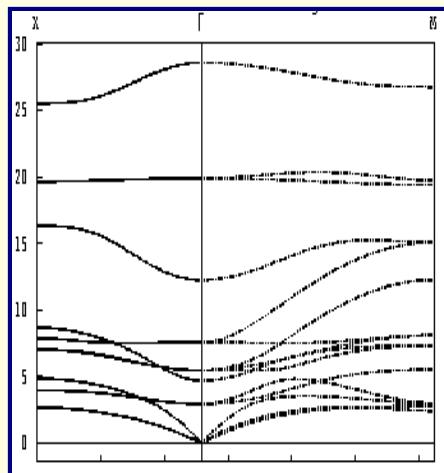
Exact:  $\Gamma, X, M, R$

$3 \times 3 \times 3$



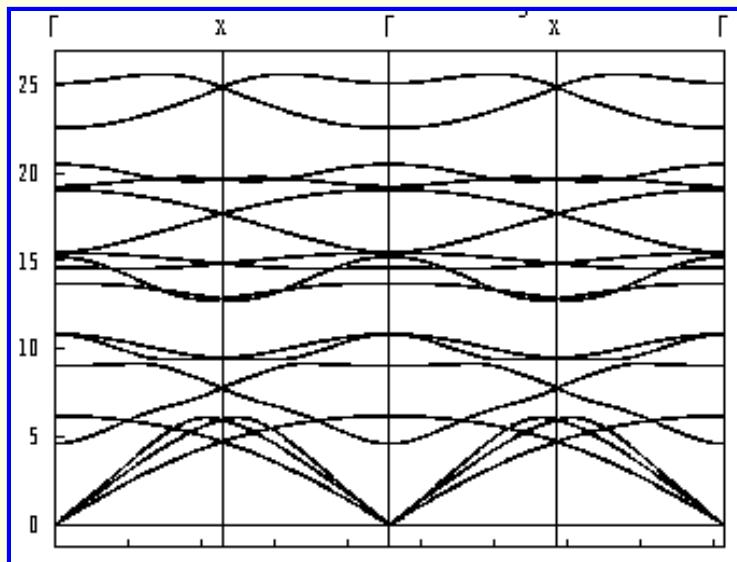
Exact:  $\Gamma$

$X \quad \Gamma \quad M$

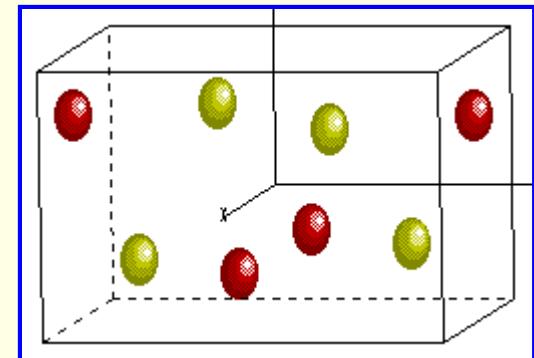


# Phonon dispersions + density of states

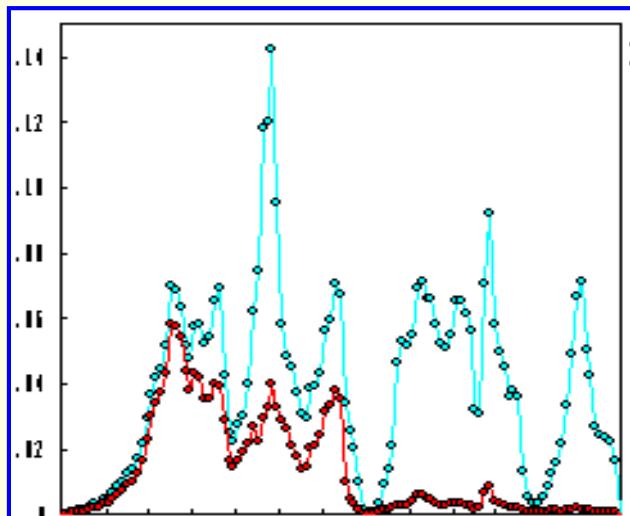
Frequency  
 $\omega$



GeO<sub>2</sub> P4\_2/mnm

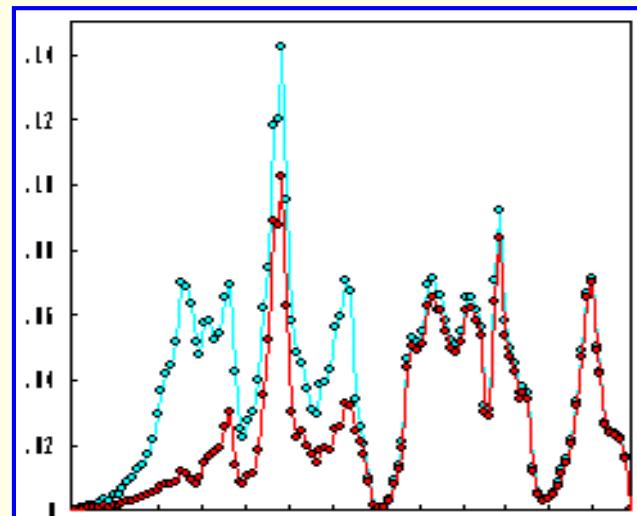


Total + Germanium



$\omega$

Total + Oxygen



$\omega$

Internal energy:

$$E = \frac{1}{2} r \int_0^\infty d\omega g(\omega) (\hbar\omega) \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$

Free energy:

$$F = rk_B T \int_0^\infty d\omega g(\omega) \ln \left[ 2 \sinh \left( \frac{\hbar\omega}{2k_B T} \right) \right]$$

Entropy:

$$S = rk_B \int_0^\infty d\omega g(\omega) \left\{ \left( \frac{\hbar\omega}{2k_B T} \right) \left[ \coth \left( \frac{\hbar\omega}{2k_B T} \right) - 1 \right] - \ln \left[ 1 - \exp \left( -\frac{\hbar\omega}{k_B T} \right) \right] \right\}$$

Heat capacity C<sub>v</sub>:

$$C = rk_B \int_0^\infty d\omega g(\omega) \left( \frac{\hbar\omega}{k_B T} \right)^2 \frac{\exp(\frac{\hbar\omega}{k_B T})}{\left[ \exp(\frac{\hbar\omega}{k_B T}) - 1 \right]^2}$$

Thermal displacements:

$$B_{ij}(\mu) = \langle U_i(\mu) U_j(\mu) \rangle$$

$$B_{il}(\mu) = \frac{\hbar r}{2M_\mu} \int_0^\infty d\omega g_{il,\mu}(\omega) \frac{1}{\omega} \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$

## ■ PHONON

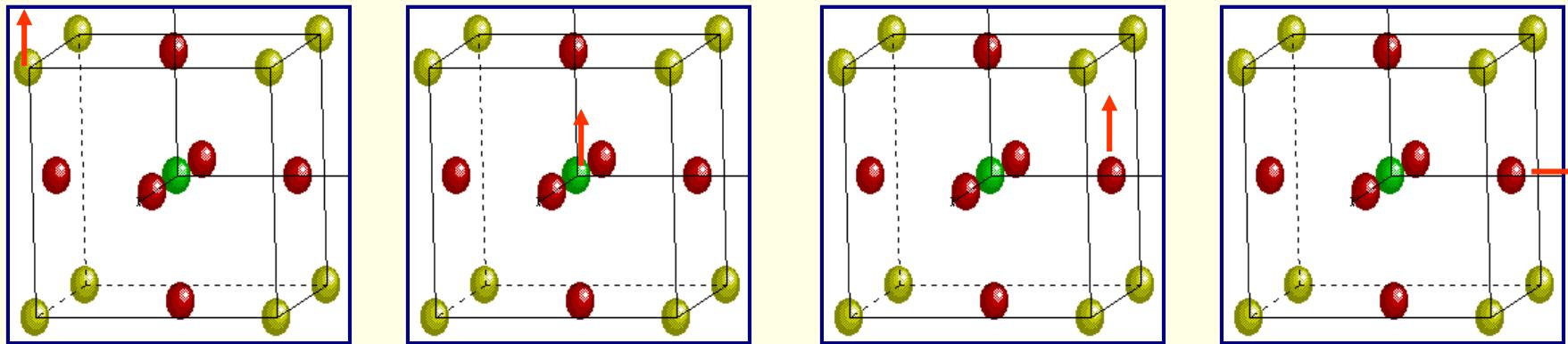
- by K.Parlinski (Crakow)
  - Linux or MS-windows
  - uses a „*direct*“ method to calculate *Force-constants* with the help of an *ab initio* program
  - with these *Force-constants phonons* at arbitrary *k-points* can be obtained
- 
- Define your spacegroup
  - Define all atoms



<https://computingformaterials.com>

# Phonons:

- selects symmetry adapted atomic displacements (*4 displacements in cubic perovskites*)



(Displacement pattern for cubic perovskite)

- select a supercell: (*e.g. 2x2x2 atom P-type cell*)
  - calculate all *forces* for these *displacements* with high accuracy (*WIEN2k*)
- 
- → *force constants* between all atoms in the supercell
  - → *dynamical matrix* for arbitrary *q*-vectors
  - → *phonon-dispersion* ("bandstructure") using *PHONON* (*K.Parlinski*)

- Define an interaction range (supercell)

- *create displacement file*
  - *transfer case.d45 to Unix*

- Calculate forces for all required displacements

- *init\_phonon\_lapw*
    - for each displacement a **case\_XX.struct** file is generated in an extra directory
    - runs **nn** and lets you define **RMT** values like:
      - 1.85 1-16

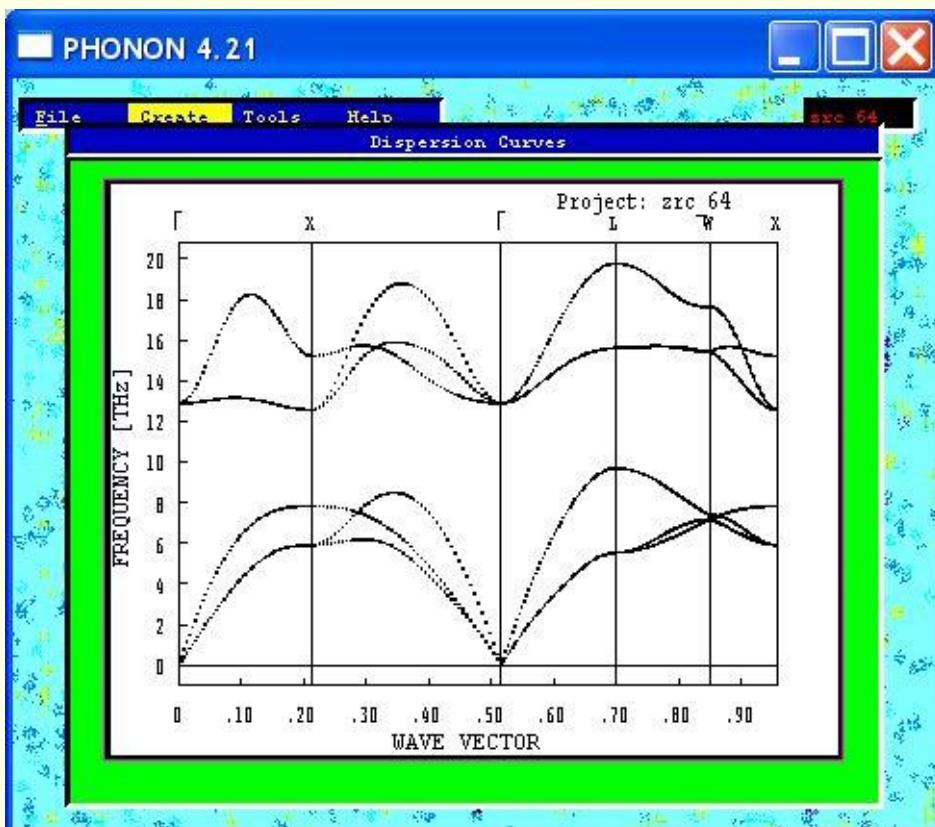


- **init\_lapw:** either **without symmetry** (and then copies this setup to all **case\_XX**)  
or **with symmetry** (must run **init\_lapw** for all **case\_XX**) (Do **NOT** use **SGROUP**)
- **run\_phonon:** **run\_lapw -fc 0.1 -i 40** for each **case\_XX**

## ■ analyze\_phonon\_lapw

- reads the *forces* of the scf runs
- generates „Hellman-Feynman“ file *case.dat* and a „symmetrized HF-file *case.dsy* (when you have displacements in both directions)
  - check quality of forces:
  - sum  $F_x$  should be small (0)
  - $\text{abs}(F_x)$  should be similar for +/- displacements

- transfer *case.dat* (*dsy*) to Windows
- Import HF files to PHONON
- Calculate force constants
- Calculate phonons, analyze phonons eigenmodes, thermodynamic functions



# Applications:

- phonon frequencies (compare with IR, raman, neutrons)
- identify dynamically unstable structures, describe phase transitions, find more stable (low T) phases.
- free energies at  $T > 0$ ; quasiharmonic approximation

Pyrochlore structure of  $\text{Y}_2\text{Nb}_2\text{O}_7$ : strong phonon instabilities  $\rightarrow$  phase transition

