

NMR: chemical and Knight shifts



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NMR shielding





Shielding of applied B-field leads to material dependent changes in transition energy





































for insulator:







chemical shift: $\delta(ppm) = (\sigma_{ref} - \sigma) \times 10^6$







 The induced magnetic field *B_{orb}* is derived from induced current *j* using the Biot-Savart law

$$B_{orb}(r) = \frac{1}{c} \int j(r') \times \frac{r - r'}{|r - r'|^3} d^3r'$$

• Current *j*(*r*) comes from DFT:

$$j(r') = \sum_{o} \langle \Psi_o | J(r') | \Psi_o \rangle$$
wave function in presence of B current operator





Linear response theory

Wavefunction in first-order perturbation

$$|\Psi_o\rangle = |\Psi_o^{(0)}\rangle + |\Psi_o^{(1)}\rangle$$
 first of the o

first order perturbation of the occupied states

$$\begin{split} &|\Psi_o^{(1)}\rangle = \sum_e |\Psi_e^{(0)}\rangle \frac{\langle \Psi_e^{(0)} | H^{(1)} | \Psi_o^{(0)}\rangle}{\epsilon_{\rm o} - \epsilon_e} \end{split}$$

 $p \rightarrow p + A(r')$ where $A(r) = \frac{1}{2}B \times (r - d)$ (symmetric gauge)

$$H^{(1)} = \frac{1}{2c} \boldsymbol{L} \cdot \boldsymbol{B} = \frac{1}{2c} \boldsymbol{r} \times \boldsymbol{p}.\boldsymbol{B}$$

ill defined for extended systems





$$\mathbf{r} \cdot \hat{\mathbf{u}}_i = \lim_{q \to 0} \frac{1}{2q} \left(e^{iq\hat{\mathbf{u}}_i \cdot \mathbf{r}} - e^{-iq\hat{\mathbf{u}}_i \cdot \mathbf{r}} \right)$$

H⁽¹⁾ couples *k* and *k*±*q* states

 Eigenfunctions have to be computed on k-meshes shifted by ±*q* for small q

APW description of unoccupied states:



 APW basis is perfect only for states with eigen energy close to linearization energy

 to remedy this we include extended set of local orbitals (NMR LO)



- NMR LO has has node at the sphere boundary
- Number of nodes increase by one in subsequent LO





 APW does not include directly radial derivative of u(r) which results in slow convergence with respect to number of NMR LO
 Adding r*du/dr radial functions to the basis helps

$$\xi_{l,k}(r,\tilde{\epsilon}) = \begin{cases} r\frac{d}{dr}u_{l+1}(r,\tilde{\epsilon}) + (l+2)u_{l+1}(r,\tilde{\epsilon}), & k = 1\\ r\frac{d}{dr}u_{l-1}(r,\tilde{\epsilon}) - (l-1)u_{l-1}(r,\tilde{\epsilon}), & k = 2 \end{cases}$$

$$\tilde{u}_{l,k}(r) = \xi_{l,k}(r,\tilde{\epsilon}) - \sum_{i} b_{l,k,i} u_{l,i}(r).$$

$$|\phi_{lm,k}\rangle = \tilde{u}_{l,k}(r)Y_{lm}$$

$$\mathcal{G}(\epsilon_i) = \sum_e \frac{|\Psi_e^{(0)}\rangle \langle \Psi_e^{(0)}|}{\epsilon_i - \epsilon_e} + \sum_k \frac{|\phi_k\rangle \langle \phi_k|}{\langle \phi_k| (\epsilon_i - H) |\phi_k\rangle}$$





 Core states are covered by a separate eigenvalue problem, contribution is purely diamagnetic:

$$\mathbf{j}_{ind}(\mathbf{r}') = -\frac{1}{2c}\rho_{core}(\mathbf{r}')\mathbf{B}\times\mathbf{r}'$$

 Separate treatment of core and valence orbitals introduces some errors, corrected by:

$$\begin{aligned} |\Psi_{o}^{(1)}\rangle &= \sum_{e} |\Psi_{e}^{(0)}\rangle \frac{\langle \Psi_{e}^{(0)} | H^{(1)} | \Psi_{o}^{(0)} \rangle}{\epsilon_{o} - \epsilon_{e}} \\ &+ \sum_{core} |\Psi_{core}^{(0)}\rangle \frac{\langle \Psi_{core}^{(0)} | H^{(1)} | \Psi_{o}^{(0)} \rangle}{\epsilon_{o} - \epsilon_{core}} \leftarrow \text{Correction} \end{aligned}$$

PRB 89, 014402 (2014)





• NMR shielding σ of the Ar atom:



R.Laskowski, P.B., PRB, 89 (2014), 014402



Test of method





j(r) from perturbation theory does not agree with exact current (small *r*)
 we need basis set improvements for unoccupied states (high E-LOs + "Sternheimer" (DUC) basis functions) and core-corrections

Theory – Orbital fields in solids:



In insulators the induced **magnetic field** (B_{ind}) is obtained from the induced orbital current (*j*_{ind}) using **Biot-Savart's** law:

$$\mathbf{B}_{ind}(\mathbf{r}) = \frac{1}{c} \int d^3 \mathbf{r}' \, \mathbf{j}(\mathbf{r}') \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}$$

in DFT the **current density** *j*(*r*) can be obtained from :

 $\mathbf{j}_{\text{ind}}(\mathbf{r}') = \sum_{\sigma} \left[\left\langle \Psi_o^{(1)} \middle| \mathbf{J}^{(0)}(\mathbf{r}') \middle| \Psi_o^{(0)} \right\rangle + \left\langle \Psi_o^{(0)} \middle| \mathbf{J}^{(0)}(\mathbf{r}') \middle| \Psi_o^{(1)} \right\rangle \text{ paramagnetic} \right]$

+ $\langle \Psi_o^{(0)} | \mathbf{J}^{(1)}(\mathbf{r}') | \Psi_o^{(0)} \rangle$], diamagnetic contributions

perturbed w.f. $\Psi^{(1)}$ is obtained from perturbation theory

reens function approach

H⁽¹⁾ is the **external magnetic field** in symmetric gauge

$$H^{(1)} = \frac{1}{2c}\mathbf{r} \times \mathbf{p} \cdot \mathbf{B}$$

the magnetic field breaks periodic boundary conditions

 \rightarrow the *r* operator is ill defined

pioneered by F.Mauri et al. "GIPAW" method

$$\mathbf{r} \cdot \hat{\mathbf{u}}_i = \lim_{q \to 0} \frac{1}{2q} (e^{iq\hat{\mathbf{u}}_i \cdot \mathbf{r}} - e^{-iq\hat{\mathbf{u}}_i \cdot \mathbf{r}})$$





Induced current in LAPW



Induced current field for BaO (fcc) , **B**_{ext} in (001)



$$\mathbf{j}_{ind}(\mathbf{r}) = \begin{cases} \sum_{\mathbf{G}} \mathbf{j}_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{r}}, & \mathbf{r} \in I\\ \sum_{l,m} \mathbf{j}_{l,m}^{\alpha}(r) Y_{l,m}(\hat{r}), & \mathbf{r} \in S_{\alpha} \end{cases}$$







bulk oxides or fluorides (O and F-NMR)

• VASP agrees very well with WIEN2k when optimized potentials are used





Chemical shift of ³³S







position/occupation of Me-d states determines CS









- Master script: x_nmr [options]
- 1) run SCF calculation

2) prepare case.in1_nmr (add NMR LO): x_nmr -mode in1
case.in1_nmr: (no HDLOs in case.in1 !!)

- WFFIL EF=.533144859350 (WFFIL, WFPRI, ENFIL, SUPWF)
- 7.00 10 4 (R-MT*K-MAX; MAX L IN WF, V-NMT
- 0.30 19 0 (GLOBAL E-PARAMETER WITH n
- 0 -0.58576 0.002 CONT 1
- 0 4.80000 0.000 CONT 1
- 0 36.60000 0.000 CONT 1
- 0 66.66000 0.000 CONT 1
- 0 104.26000 0.000 CONT 1
- 0 149.26000 0.000 CONT 1
- 0 201.50000 0.000 CONT 1







3) run x_nmr: does the following steps for you:

- 1) computes eigenvectors using shifted and non-shifted meshes, the results are stored in: ./nmr_q0, ./nmr_mqx, ./nmr_pqx, ./nmr_mqy, ./nmr_pqy, ./nmr_mqz, ./nmr_pqz (x_nmr -mode lapw1)
- 2) computes weights for each k-mesh, (x_nmr -mode lapw2)
- 3) computes core wave-functions (x_nmr -mode lcore)
- 4) computes induced current (x_nmr -mode current)
- 5) integrates the current ((x_nmr -mode integ)

produces: case.output_integ

- :NMRTOT001 ATOM: Ba1 1 NMR(total/ppm) Sigma-ISO = 5384.00 Sigma_xx = 5474.82 Sigma_yy = 5385.93 Sigma_zz = 5291.24
- :NMRASY001 ATOM: Ba1 1 NMR(total/ppm) ANISO (delta-sigma) = -139.13 ASYM (eta) = 0.958 SPAN = 183.57 SKEW =-0.032

• ...

- x_nmr -h prints help
- x_nmr –p runs in parallel





band wise analysis

x_nmr -emin e1 -emax e2

 character analysis (s,p,d) of the wave functions of occupied and empty states

 $\mathbf{x}_{nmr} - \text{filt}_{curr} o \ atom I \qquad \mathbf{x}_{nmr} - \text{filt}_{curr} \text{fop} \ atom I$ $\mathbf{j}_{ind}(\mathbf{r}') = \frac{1}{c} \sum_{o} Re \left[\langle \Psi_{o}^{(0)} | \mathbf{J}^{0}(\mathbf{r}') | \tilde{\Psi}_{o}^{(1)} \rangle \right]$

$$\begin{split} \mathbf{x}_{nmr} - \text{filt}_{cxyz}_{q} \ atom I & \mathbf{x}_{nmr} - \text{filt}_{cxyz}_{o} \ atom I \\ |\tilde{\Psi}_{o}^{(1)}\rangle = \sum_{e} |\Psi_{e}^{(0)}\rangle \frac{\langle \Psi_{e}^{(0)}| \left[(\mathbf{r} - \mathbf{r}') \times \mathbf{p} \cdot \mathbf{B} \right] |\Psi_{o}^{(0)}\rangle}{\epsilon_{\circ} - \epsilon_{e}} \\ \text{large } \delta \mathbf{E} \rightarrow \text{small effect} \end{split}$$



Origin of shielding in fluorides







NMR shielding at fluorine nucleus in alkali fluoride series for different couplings

+320 ppm from F-1s,2s (constant)

PRB 85, 245117 (2012)

Schematic diagram representing major couplings contributing to NMR shielding





room T: tetragonal phasehigh-T cubic phase:

10⁻⁴ S/cm, 2 orders better cond.
 than tetragonal phase

 cubic LLZO by Al³⁺-doping
 A^{β+} replaces Li⁺, but crystallographic site is unknown

²⁷Al NMR: 2 signals, 14 ppm apart 4-fold coordinations ??

 neutron diffraction: Al in 6-fold coord.

Figure 1. Crystal structure of cubic LLZO. The yellow dodecahedrally coordinate La^{3+} (at the Wyckoff position 24*c*) and orange octahedrally coordinate Zr^{4+} (16*a*). The blue spheres correspond to tetrahedrally coordinated (24*d*) Li⁺, green spheres to octahedrally coordinated (48*g*) Li⁺, and red ones to distorted 4-fold coordinated (96*h*) Li⁺.

D. Rettenwander et al., Chem.Mat. 26, 2617 (2014)

 $Li_7La_3Zr_2O_{12}$











1.2 Å

0.9 Å

4-fold 24d lowest energy

followed closely by

- dist. 4-fold 96h
- 6-fold **48g**



NMR chemical shifts





shift values, $\Delta\delta$, (13 to 18 ppm) in literature. The reference point refers to $\Delta \delta = 0$ given by the calculations.





In insulators the orbital current determines the shielding

• σ it is usually small (ppm) and often diamagnetic ($B_{ind} = -\sigma B_{ext}$)

In metals there is in addition a spin current

- the shift can be much larger (%) and often the spin current dominates (paramagnet)
- the external magnetic field leads to an exchange splitting of the spin-up and dn bands and an effective spin magnetization density. This results in a hyperfine field at the nucleus (Fermi contact term) and a dipolar contribution (usually small):

$$B_{\rm hf} = \frac{8\pi}{3} \mathbf{m}_{\rm av} + \int \frac{S(r)}{r^3} [3(\mathbf{m}(r)\hat{r})\hat{r} - \mathbf{m}(r)]$$

*m*_{av}: spin density at the nucleus (averaged over r_{Thomson})

exchange splitting





→ valence (4s) hyperfine field due to direct polarization
→ core (1s,..) field due to response to induced 3d moments

even "big" magnetic fields correspond to tiny energies

■ 100 T ~ 1 mRy

numerical difficulty:

- enormous k-meshes (1.000.000 k-points in Al)
- temperature smearing (2 mRy = 300 K)



I E N 2k

comparison with experiment





very good agreement given the rather old experimental data and problems with the "reference".



experiment measures vs. a reference compound (sometimes liquids) $\delta_{th} = \sigma_{ref} - (\sigma_o + \sigma_s)$

	σ	$\sigma_{o(ref)}$	σ	δ _{th}	δενη	$\delta_{th} = \delta_{exp}$	
Li /Li ₂ O	81	96	-264	279	260	19	
Na / NaBr	518	551	-1021	1054	1070	-16	
K / KBr	1126	1153	-2560	2589	Often it	is argued t	hat the
Rb / RbCl	3031	3028	-6826	6822	orbital	shifts of m	etal
Cs / CsCl	5473	5380	-16177	16083	and refe	erence car	icel
Mg / MgCl ₂	505	552	-1078	1124	and only	the Knight	(spin)
Ba / BaCl ₂	5730	5661	-4160	4092	shift remains		
AI / AIPO ₄	519	512	-1591	1584	Shine Ferr		
$In / In_2(SO_4)_3$	2807	3676	-8012	8881	This is tr	rue only for	sp-
V / NaVO ₃	-5988	-1453	488	4046	elemen	ts	SP
Cr / Na ₂ CrO ₄	-9847	-2567	461	6818	cicilien		
Mo / K ₂ MoO ₄	-5795	-825	-27	4997	υτυυ	-TTOO	
Cu / CuBr	-330	492	-1568	2390	2380	10	
Ag / AgNO ₃	2219	3772	-3670	5223	5210	13	1%



experiment measures vs. a reference compound (sometimes liquids)

 $\delta_{\rm th} = \sigma_{\rm ref} - \sigma_{\rm o} - \sigma_{\rm s}$

	σ_{o}	$\sigma_{o(ref)}$	σ_{s}	δ_{th}	δ_{exp}	$\delta_{\sf th}$ - $\delta_{\sf exp}$	
Li /Li ₂ O	81	96	-264	279	260	19	
Na / NaBr	518	551	-1021	1054	1070	-16	
K / KBr	1126	1153	-2560	2589	Why is t	he Kniaht ((snin)
Rb / RbCl	3031	3028	-6826	6822	shift not always strong		onaly
Cs / CsCl	5473	5380	-16177	16083	narama	anetic	ongry
Mg / MgCl ₂	505	552	-1078	1124	(negative	P) 777	
Ba / BaCl ₂	5730	5661	-4160	4092	(negativ	c)	
AI / AIPO ₄	519	512	-1591	1584	It should	l he directl	v
$In / In_2(SO_4)_3$	2807	3676	-8012	8881	proport	ional to t	י ופ
V / NaVO ₃	-5988	-1453	488	4046	valence	-s DOS at	- F_
Cr / Na ₂ CrO ₄	-9847	-2567	461	6818	Valence		╴┗╞╺
$Mo / K_2 MoO_4$	-5795	-825	-27	4997	υτυυ	-1102	
Cu / CuBr	-330	492	-1568	2390	2380	10	
Ag / AgNO ₃	2219	3772	-3670	5223	5210	13	





- The induced magnetic moment of "d"-character polarizes the core states (usually in opposite way).
- This core shift can be even larger than the valence shift and fully compensate it.

	$m_{\rm s} (10^{-3} \mu_{\rm B})$			$\sigma_{\rm s}~({\rm ppm})$		
	s	Р	d	valence	core	
Li	0.4	1.1	0.0	264	0	
Na	0.6	0.5	0.1	1033	-12	
K	0.4	0.2	0.0	2556	4	
Rb	0.4	0.2	0.2	6795	31	
Cs	0.5	0.2	0.7	16084	93	
Mg	0.5	0.7	0.3	1097	-16	
Ba	0.0	0.3	2.1	4078	82	
Al	0.3	0.6	0.2	1584	7	
In	0.4	0.9	0.0	7956	56	
v	0.4	1.9	25.1	3439	-3927	
Cr	0.0	0.2	5.3	613	-1074	
Mo	0.0	0.2	3.1	150	-123	
Cu	0.1	0.2	0.7	1677	-109	
Ag	0.2	0.7	0.3	3708	-39	





- 1) Spin-polarized calculation with zero moment
 - instgen -nm # generate nonmagnetic atomic configurations
 - init_lapw -sp -fermit 0.004 -numk XXX ... # initialization
 - runsp_c_lapw -c 0.00001 [-p] ... # run scf with zero moment
- 2) Copy input file specifying 100T field
 - cp \$WIENROOT/SRC templates/case.vorbup(dn)_100T case.vorbup(dn)
- 3) SCF calculation with external magnetic field
 - runsp_lapw -orbc -cc 0.000001 [-p] ... # scf calculation
 - grepline :HFF0XX case.scf # get the hyperfine field in kGauss







Ternary and quaternary compounds with TT`=Ni,Cu,Ag,Au,Pd





Overall good correlation with experiment but a few exceptions:

- Al and Sc-NMR in ScNiAuAl
- Sc-NMR in ScAg₂Al



Benndorf et al., Z.anorg.allgem.Chemie 641, 168 (2015)





exp. lattice parameter disagrees with Vegards law and theoretical a₀

300 ppm shift for Al !!! correcting most of the differences





$\sigma_{\text{orb}} \text{ and } \sigma_{\text{spin}} \text{ in ScTT`Al}$





A



Sc

huge **paramagnetic** variation small **dia/paramagn**. variation determined by σ_{orb} 3d valence e⁻



Induced density and potential in ScNiAuAl





induced spin-density (100T):

Al: small and spherical symmetric Sc: large 3d-"t2g" like spin-density large induced Sc magn.moment



induced scf difference potential V_{up} –V_{dn}:

- 100T = constant 8.5 10⁻⁴ Ry
- ΔV not constant at all
- radial differences due to contraction/ expansion of spin-up/dn wf.
- non-spherical around Sc



Decomposition of Knight shift in ScTT'Al







Ga-NMR in intermetallic gallides











routine NMR calculations for Knight and Chemical shifts in solids are now possible

- huge k-meshes and Fermi broadening necessary, but manageable
- orbital shift: paramagnetic part when d-e⁻ are present/available above E_F either on the NMR-atom or its neighbor
- spin-contact term dominates in metals, but orbital part not negligible and for d elements sometimes more important than spin-part
- core-polarization due to induced 3d (4d) moments can dominate (cancel) the direct 4s (5s) valence contribution
- *important spin-dipolar term when charge asymmetry at* E_F *exists*
- "Slope"-problem = DFT-problem ?