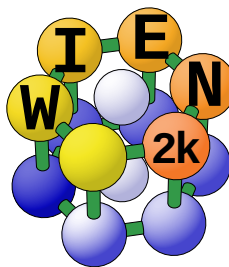


# Band-transport and thermoelectricity.

Georg Madsen

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University of Aarhus



# Overview

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- ✕ Band transport
  - ➡ electric and magnetic fields and thermal gradients
  - ➡ Semi-classic theory
  - ➡ Program: Transport
- ✕ Thermoelectrics
  - ➡ The figure of merit
  - ➡ Phonon Glass - Electron crystal
  - ➡ Skutterudites
  - ➡ Clathrates
  - ➡ Phonon DOS
- ✕ Outlook

# Transport

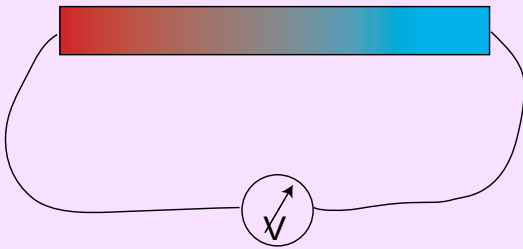
## Transport equation

$$j_i = \sigma_{ij} E_j + \sigma_{ijk} E_j B_k + \sigma_{ijkl} E_j B_k B_l + \nu_{ij} \nabla_j T + \dots$$

- $j$  current density
- $\sigma$  conductivity tensors
- $E$  electric field
- $B$  magnetic field

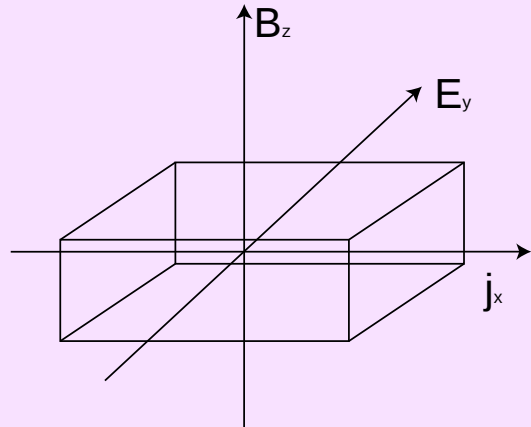
## Seebeck coefficient

$$S_{ij} = E_i (\nabla_j T)^{-1} = \sigma_{i\alpha}^{-1} \nu_{j\alpha}$$



## Hall coefficient

$$R_{ijk} = j_i^{-1} E_j B_k^{-1} = \sigma_{j\alpha}^{-1} \sigma_{i\beta}^{-1} \sigma_{\alpha\beta k}$$



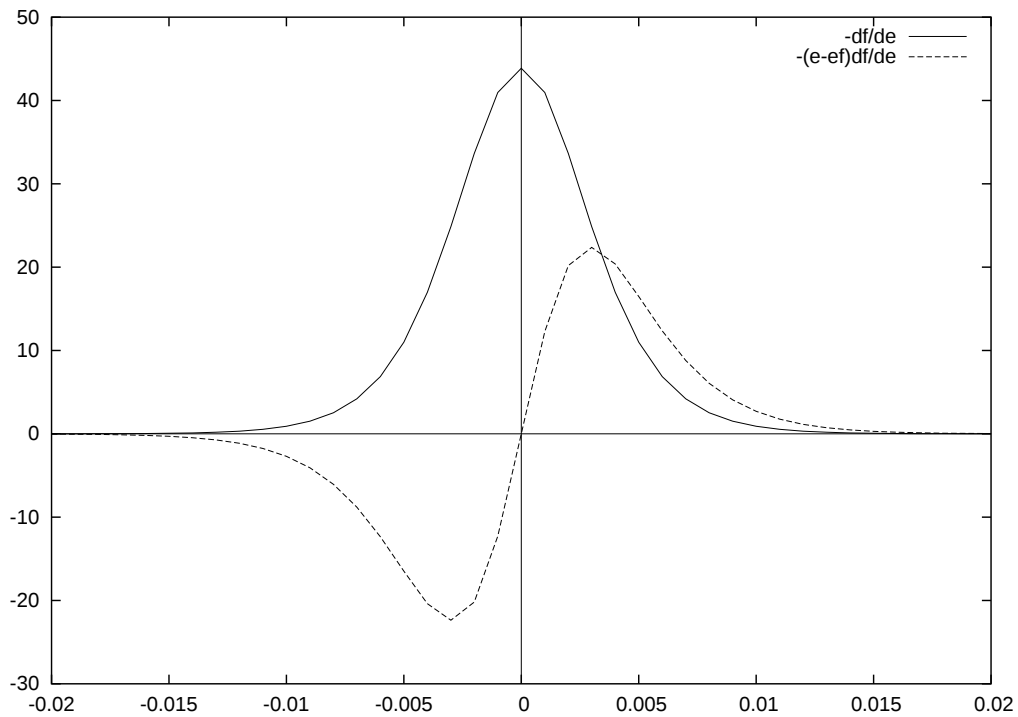
# Semi-classic approximation

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$$\sigma_{\alpha\beta} = \frac{e^2\tau}{\Omega} \sum_{\mathbf{k}} v_{\alpha}(\mathbf{k})v_{\beta}(\mathbf{k}) \left( -\frac{\partial f(\varepsilon)}{\partial \varepsilon} \right) \quad (1)$$

$$\nu_{\alpha\beta} = \frac{e^2\tau}{\Omega T} \sum_{\mathbf{k}} v_{\alpha}(\mathbf{k})v_{\beta}(\mathbf{k})(\varepsilon_{\mathbf{k}} - \mu) \left( -\frac{\partial f(\varepsilon)}{\partial \varepsilon} \right) \quad (2)$$

$$\sigma_{\alpha\beta\gamma} = \frac{e^3\tau^2}{\hbar\Omega} \sum_{\mathbf{k}} v_{\alpha}(\mathbf{k})[\mathbf{v}(\mathbf{k}) \times \nabla_{\mathbf{k}}]_{\gamma} v_{\beta}(\mathbf{k}) \left( -\frac{\partial f(\varepsilon)}{\partial \varepsilon} \right) \quad (3)$$



J. M. Ziman, *Electrons and Phonons* (Oxford Classics Series, Clarendon Press, Oxford, 2001).

# Program: Transport

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Smoothed Fourier expansion of band energies.

$$\varepsilon(\mathbf{k}) = \sum \varepsilon_m S_m(\mathbf{k}) \quad , \quad S_m(\mathbf{k}) = 1/n \sum_{\Lambda} e^{ik\Lambda R_m}$$



$$\begin{array}{ccc} FFT(\varepsilon_m) & FFT(\varepsilon_m k_{mi}) & FFT(\varepsilon_m k_{mi} k_{mj}) \\ \downarrow & \downarrow & \downarrow \\ N(\varepsilon) & N(\varepsilon) \sigma_{ij} & N(\varepsilon) \sigma_{ijk} \end{array}$$



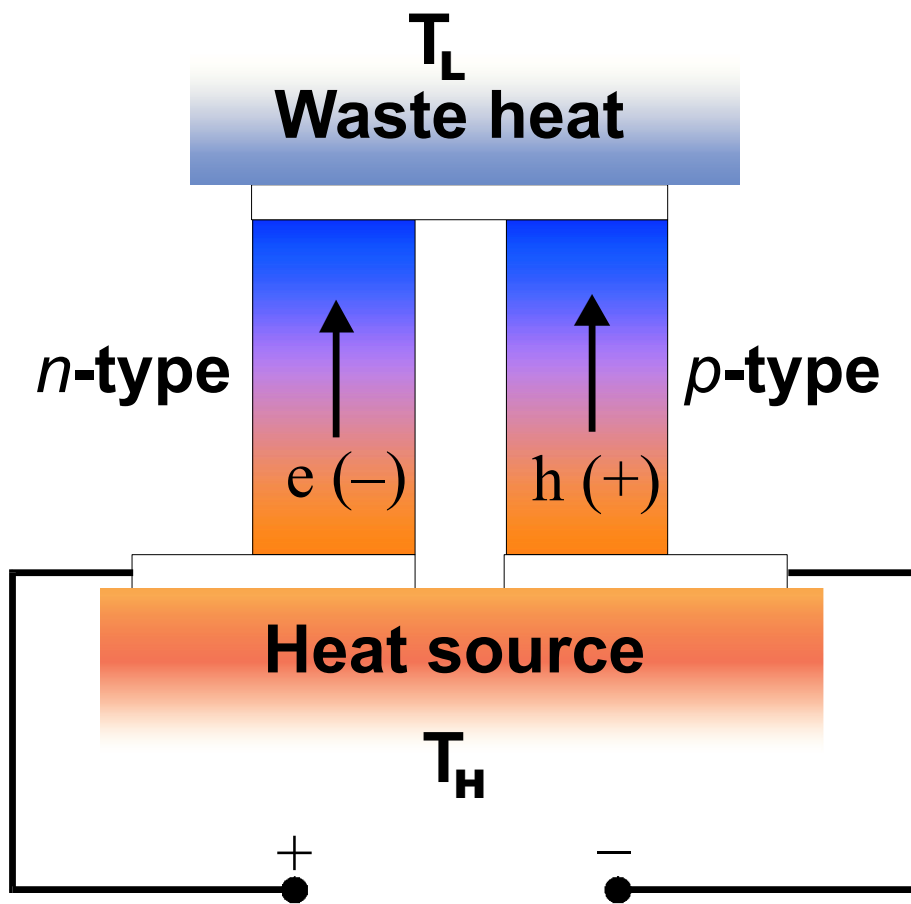
$$\sigma_{ij} = \int N(\varepsilon) \sigma_{ij}(\varepsilon) d\varepsilon$$

$$S_{ij} = \sigma_{i\alpha}^{-1} \int N(\varepsilon) \sigma_{j\alpha}(\varepsilon) (\varepsilon - \mu) d\varepsilon$$

$$R_{ijk} = \sigma_{j\alpha}^{-1} \sigma_{i\beta}^{-1} \int N(\varepsilon) \sigma_{\alpha\beta k}(\varepsilon) d\varepsilon$$

## *Figure of merit*

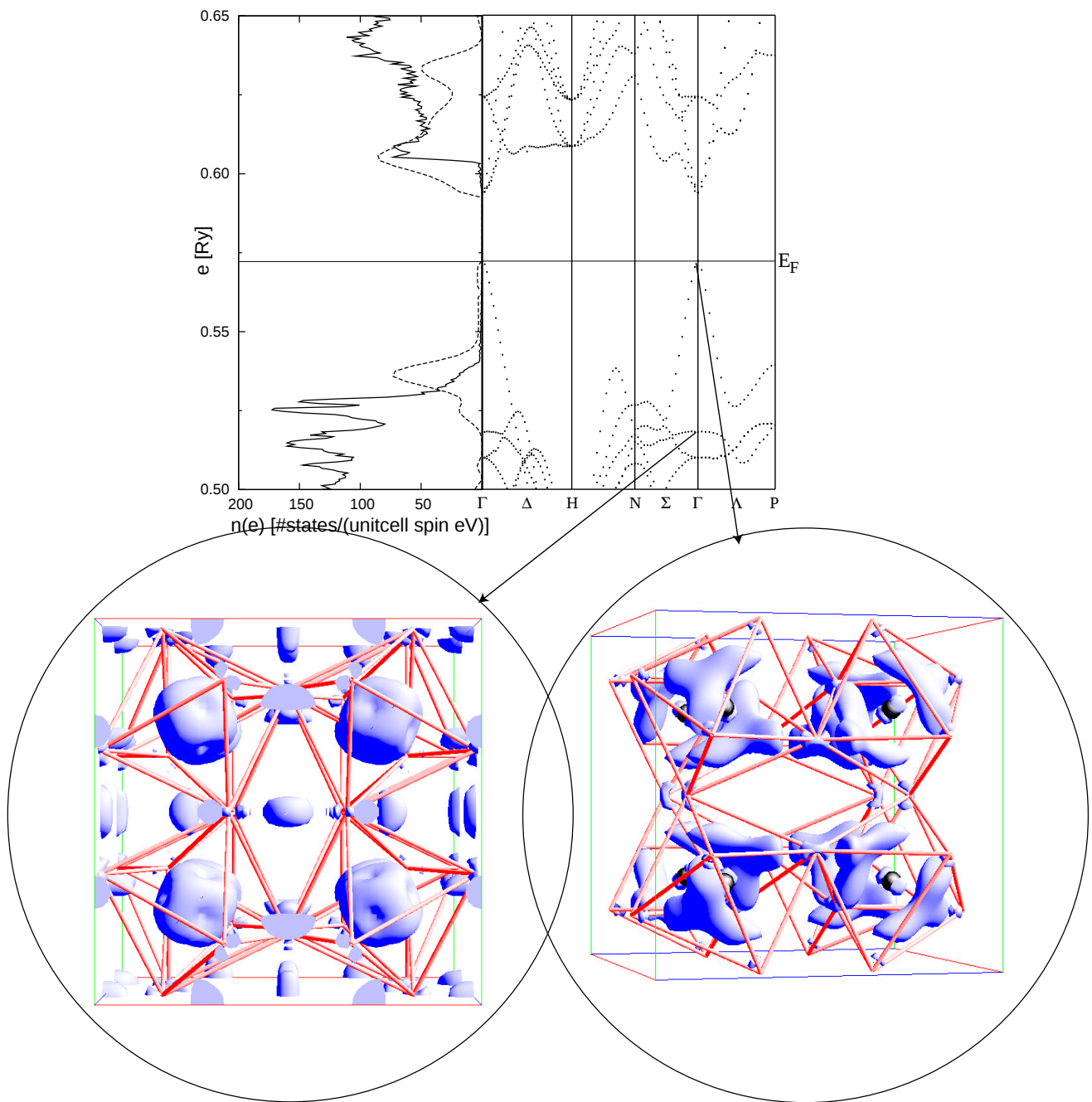
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$$ZT = (\sigma T / \kappa) S^2$$

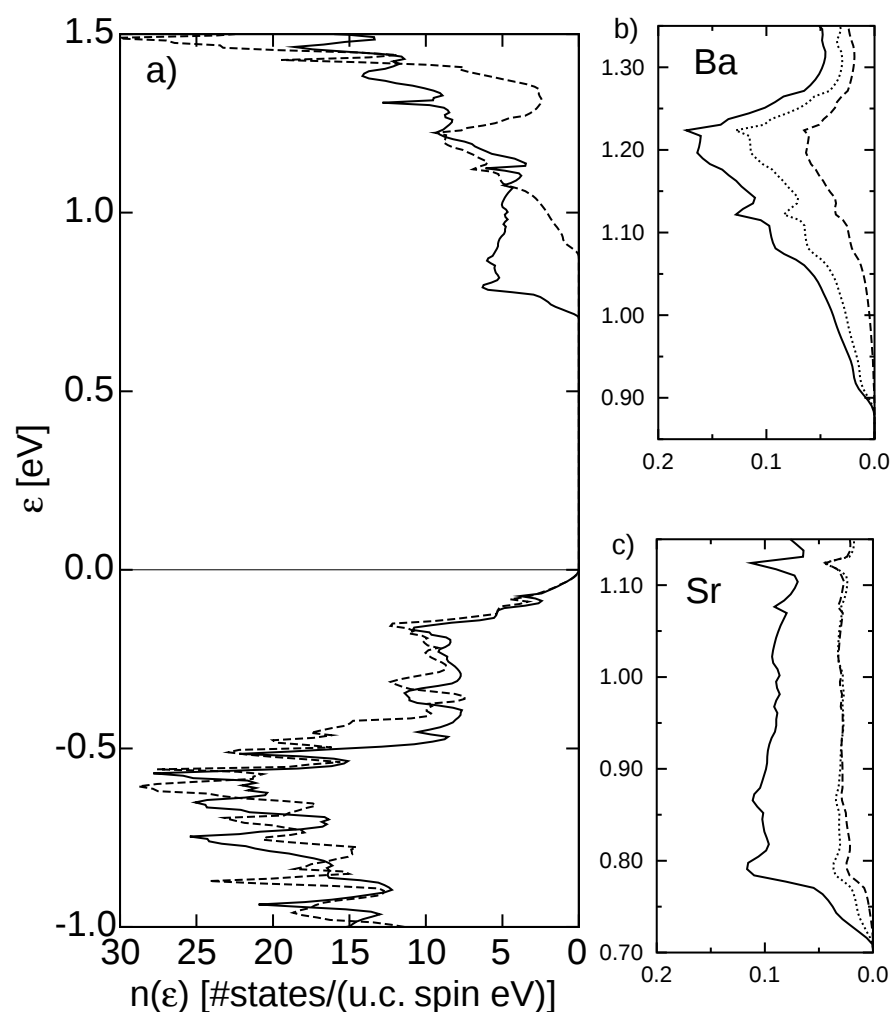
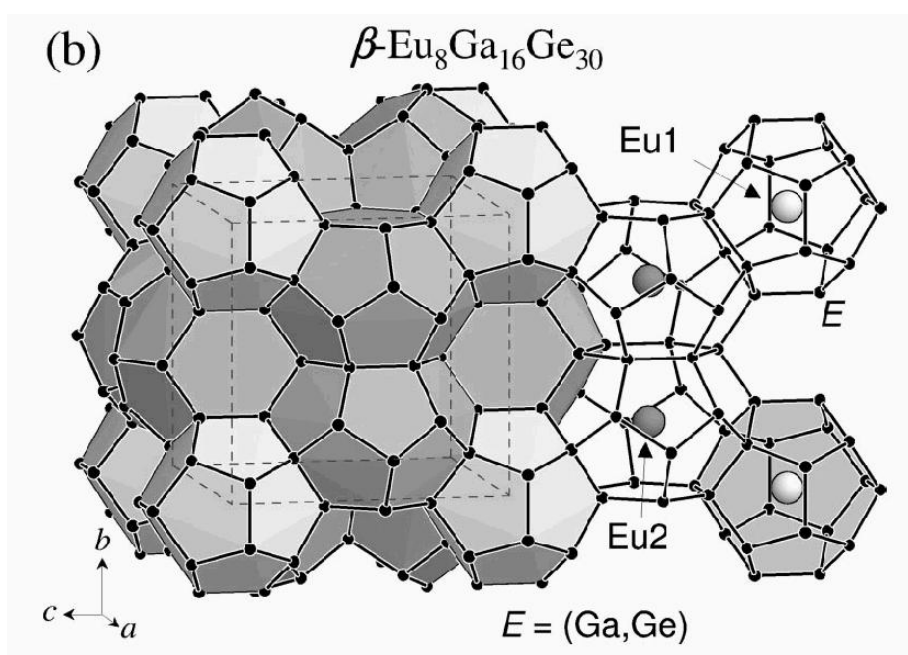
- Electrical conductivity,  $\sigma$
- Thermal conductivity (electronic and lattice),  $\kappa$
- Seebeck coefficient,  $S$

# CoSb3

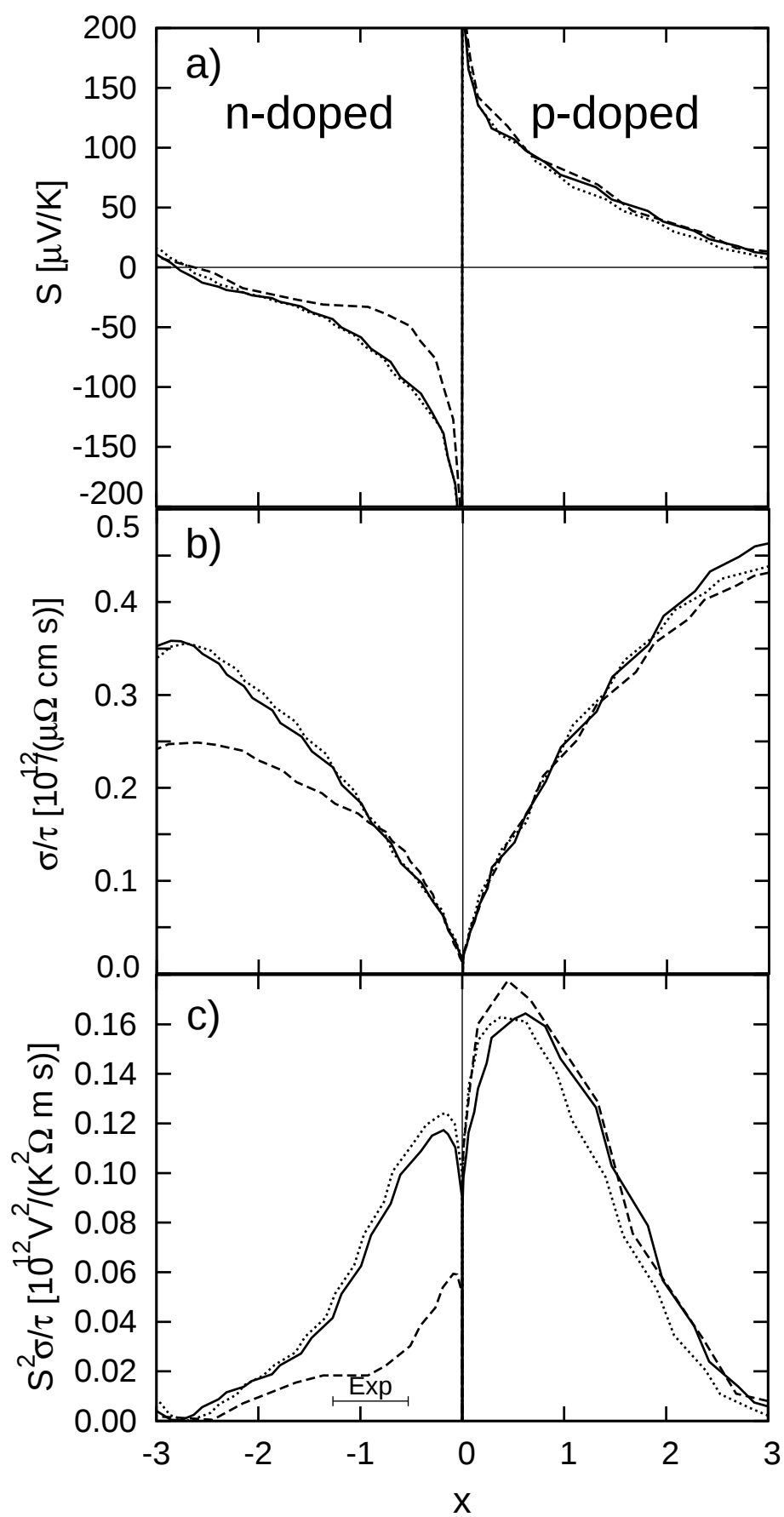


D. J. Singh and I. I. Mazin, Phys. Rev. B **56**, 1650 (1997).

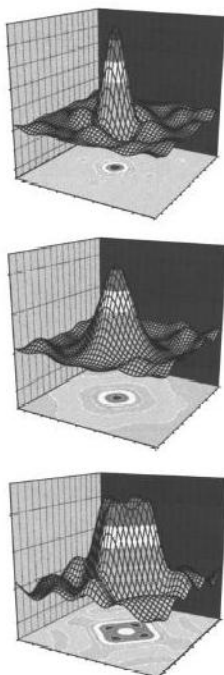
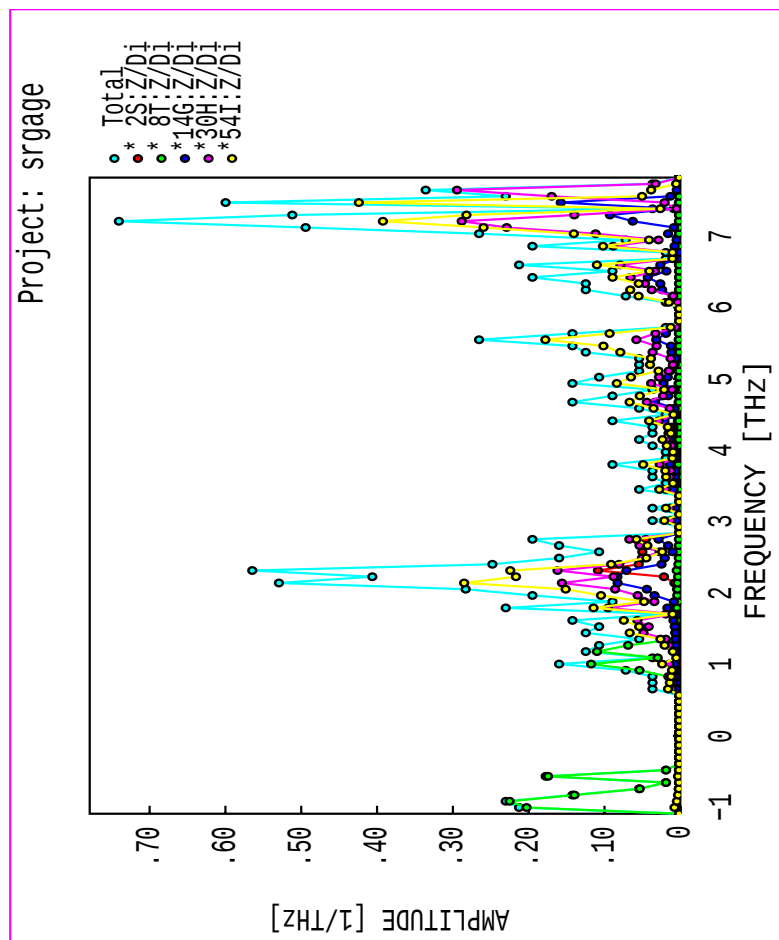
# Zintl clathrates. $(II)_8(III)_{16+x}(IV)_{30-x}$



# Type-I transport



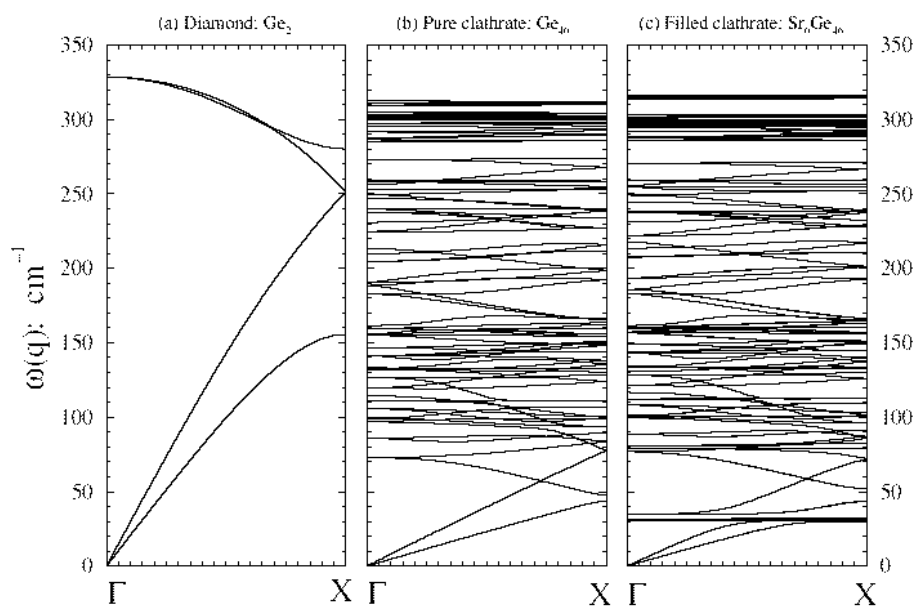
# Type-I phonon DOS



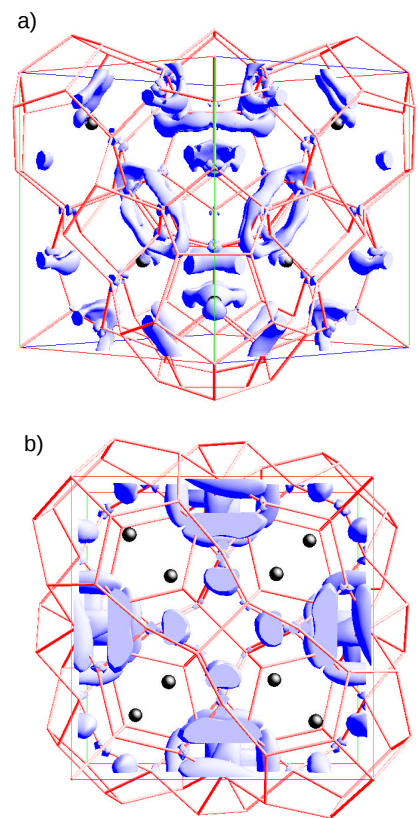
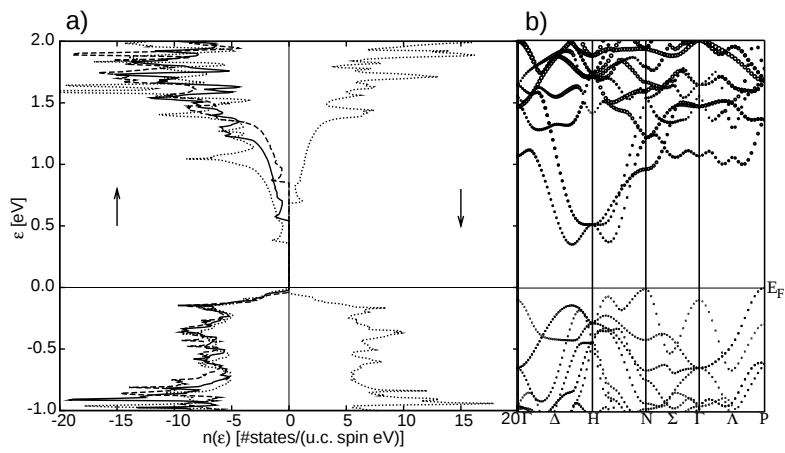
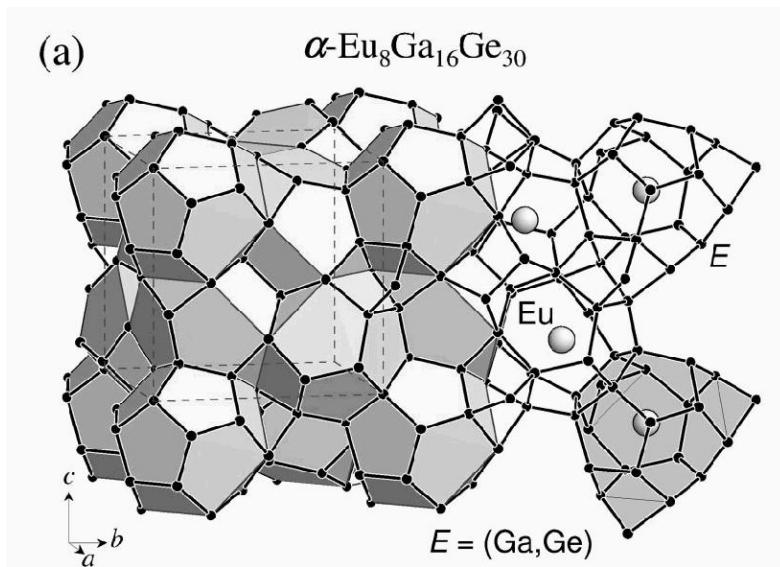
Ba

Sr

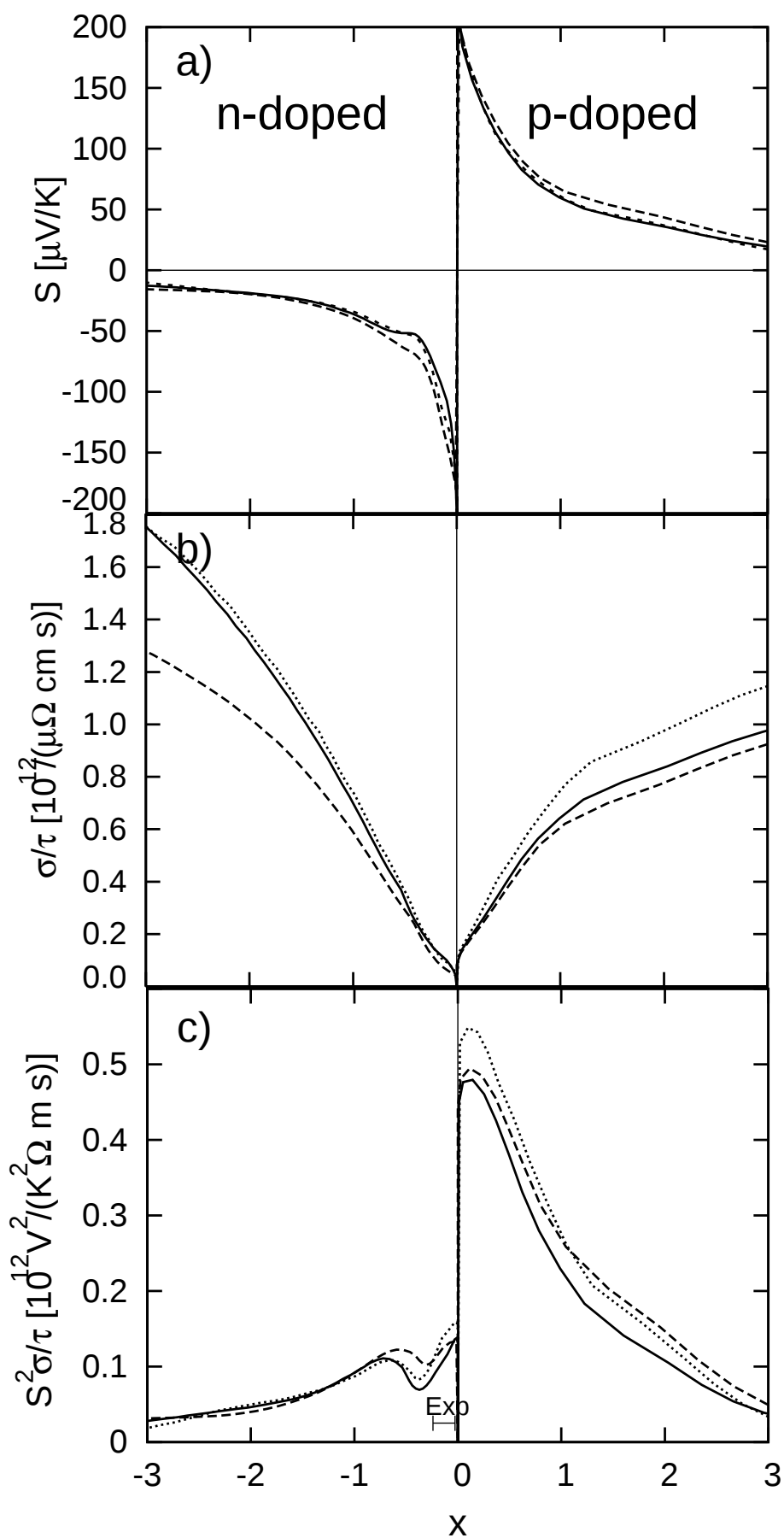
Eu



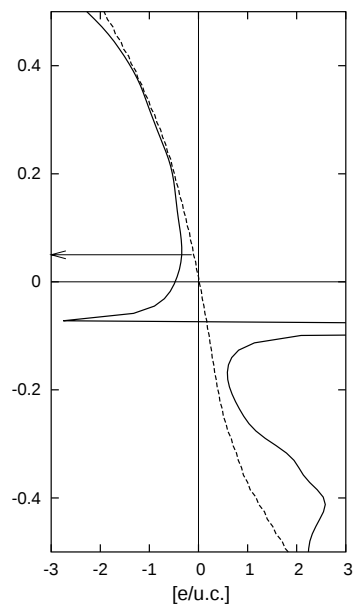
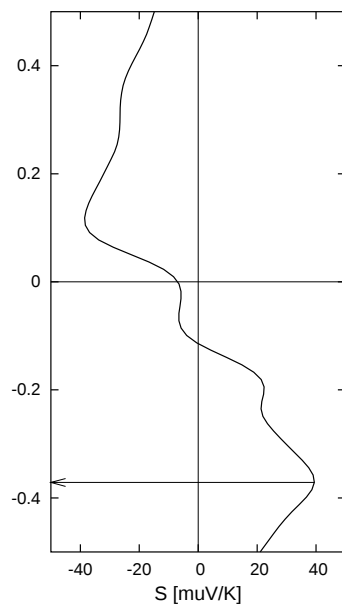
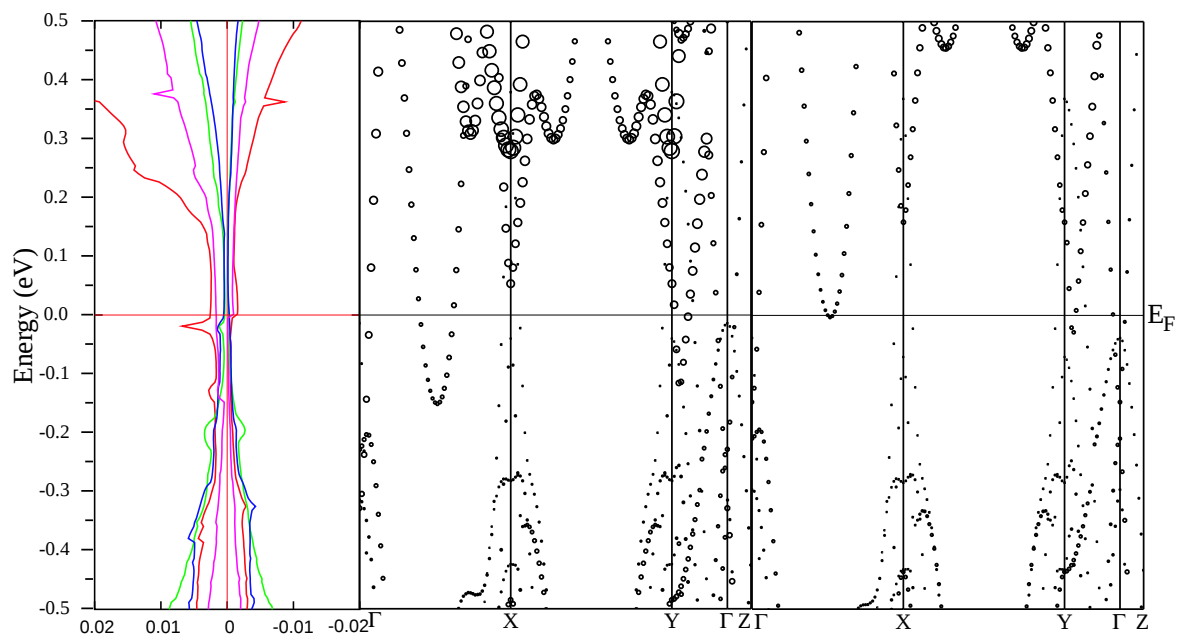
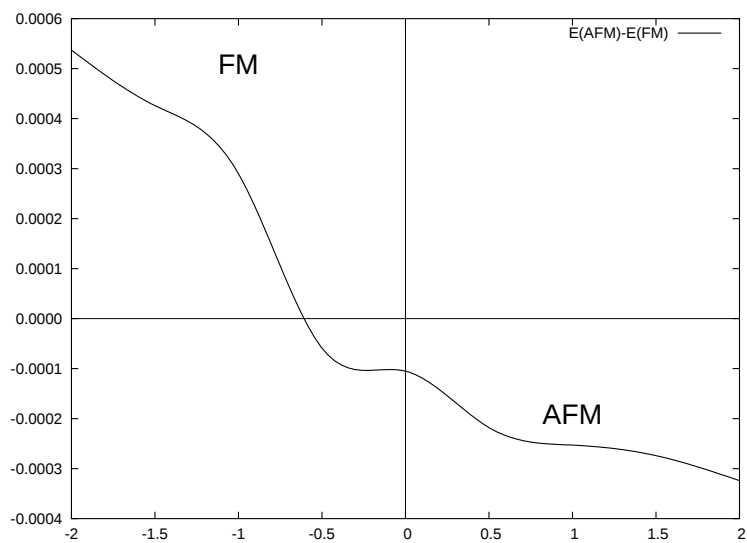
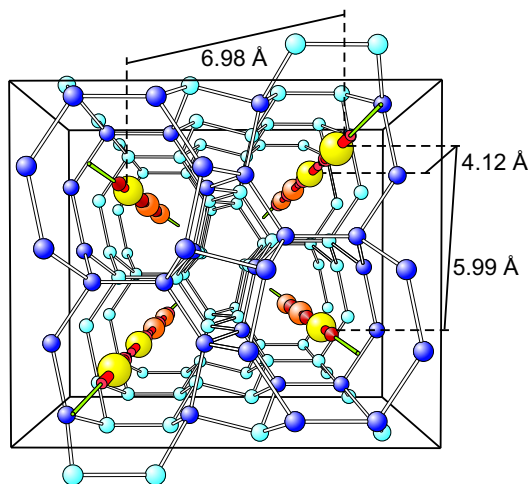
# Type-VIII



# Type-VIII transport



# Type-IV AFM clathrate



# *Acknowledgements*

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